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Abstract

This article explains the role of the convenience yield in the relationships linking spot and futures prices in commodity derivatives markets. First, this variable restores the non arbitrage relationship between the prices of the underlying asset and the derivative instrument. Second, it allows establishing connections between commodities and other assets. Third, it explains why firms store at an apparent loss. The convenience is however a controversial concept. Indeed, the absence of direct evidence for this quantity signifies, first that it is necessary to address the issue of estimating it and second, that it can be accused of being an *ad hoc* construction. Moreover, in spite of an early interest for this concept, there is no real consensus on its definition. This article aims at gathering all the reasonable explanations which were proposed trough time in the literature.

Key words

Convenience yield – Arbitrage – Commodity – Inventory – Non storable commodities

Since 1939, the convenience yield plays a crucial role in the explanation of the relationships between spot and futures prices in commodity markets. It indeed appears as a way to explain backwardation, a situation where the futures price is lower than the spot price. Although the hedging pressure theory (previously called normal backwardation theory – see box 1) provides an alternative explanation for this phenomenon, the convenience yield is particularly appealing because it gives the possibility to establish a parallel between the commodities and other derivative markets, such as the markets for currencies, interest rates and equities. On the other hand, some authors consider that the convenience yield is nothing more than a variable artificially added in order to restore the no-arbitrage relationships, with no real economic significance.

Is the convenience yield just an *ad hoc* construction, or does it have a real meaning? Is it useful for non storable commodities, like electricity or carbon emissions? This article aims to answer these questions. It is organized as follows. In a first section, it presents the role of the convenience yield in establishing a realistic relationship between spot and futures commodity prices. The second section opens the "black box": it organizes the different explanations provided through time in order to justify the use of the convenience yield. The third section is devoted to modeling issues. The fourth is centered on empirical tests, the fifth comments on the different critiques that were addressed to the convenience yield. The last section examines the new questions arising with the emergence and growth of derivative markets for non storable commodities. Indeed, the concept was initially developed for storable commodities.

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1. The “non arbitrage relationship” in commodity markets

The convenience yield is a central variable in the storage theory, which examines the information conveyed by futures prices and the discovery function of commodity derivative markets. To understand the relationships linking the spot and futures prices, this theory focuses on the reasons explaining the holding of inventories and examines the arbitrage operations undertaken by the operators between the physical and futures markets. In this theoretical framework, the role of inventory is crucial. Stocks indeed have a buffering effect: they absorb prices fluctuations. They also avoid disruptions in the flow of goods and services provided to the consumers. Lastly, they insure a link between the present and the future, allowing thus the inter-temporal allocation of the resource.

Relying on stocks and arbitrage operations, the explanation of contango situations, where the futures price is higher than the spot price, is quite straightforward. The spread between futures and spot prices is related to the cost of holding the commodity over time:

$$F(t,T) - S(t) = CS(t,T) \qquad [1]$$

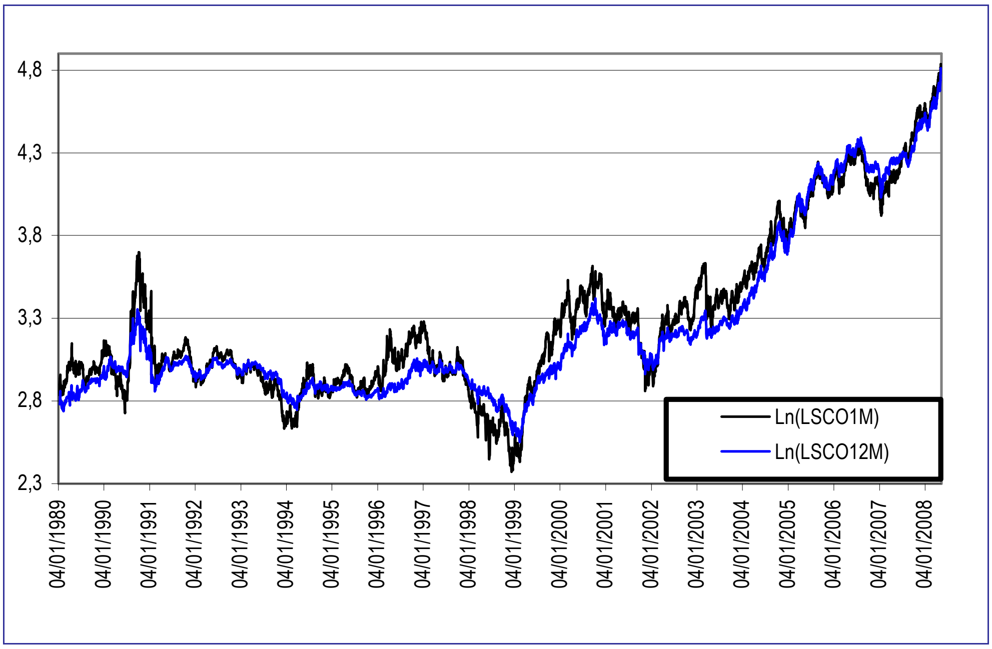
where:
- F(t,T) is the futures prices at t for delivery at T,
- S(t) is the spot price at t,
- CS(t,T) represents the storage costs (carrying charges).

The storage costs stand for fixed and variable costs. The fixed costs are due to insurance and warehouse expenses. They remain constant as long as the storage capacities are not saturated. The variable costs are due to deterioration and obsolescence, to the necessity to finance inventories and to maintenance expenses.

In the presence of surplus stocks, the level of contango can not stay higher than the storage costs. Whenever such a situation occurs, cash and carry arbitrages restore the equilibrium: it becomes profitable to buy physical stocks in the spot market, to carry them and simultaneously to sell futures contracts. These sales lead to a decrease in the futures price, whereas the spot price increases as a result of inventory purchases. Finally, arbitrage opportunities disappear. When there are surplus stocks in the physical market, backwardation situations, where the spot price is higher than the futures price, are also impossible. In such a case, it would be possible to undertake a reverse cash and carry, namely to sell the stocks on the spot market and to purchase contracts on the futures market. The multiplication of such arbitrages would lead to a fall of the spot price, resulting from massive sales of physical stocks. Simultaneously, the futures price would rise under the influence of contracts’ purchases. These operations would stop when the difference between the futures and the spot prices becomes equal to the storage cost.

Naturally, in this theoretical setting, a question arises: how is it possible to observe prices’ spreads inferior to full carrying charges? Some markets indeed are characterized by a persistent backwardation. It is a well-known phenomenon in the case of the crude oil market, as reported, for example, by Litzenberger and Rabinowitz (1995). It is illustrated by Figure 1. The latter represents the prices of the light sweet crude oil futures contracts negotiated on the New York Mercantile Exchange (Nymex), for two different maturities: 1 month (in black) and 12 months (in blue). Due to the high fluctuations recorded from 1989 to 2008 the futures prices, which are quoted in dollars per barrel, are expressed in logarithm. The figure shows that in almost 20 years, the marked witnessed only three contango situations: in 1993-1994, in 1998, and more recently in 2005-2006.

Figure 1. Light sweet crude oil futures prices (LSCO), Nymex, 1989-2008



According to Nicholas Kaldor' (1939-1940), backwardation is possible because stocks of all goods possess a yield: the convenience yield. Let us at first define this variable as the remuneration associated with the holding of physical stocks. Let us also assume that it is all the more important that physical stocks are rare – we will provide more explanations in the next section. If we introduce the convenience yield in equation [1], the prices' spread (also called the basis) becomes:

$$F(t,T) - S(t) = CS(t,T) - Cy(t,T) \qquad [2]$$

where Cy(t,T) stands for the convenience yield[2].

When the convenience yield becomes higher than storage costs, there is backwardation.

Such a characterization of the relationship between spot and futures prices is consistent with the no-arbitrage valuation of futures contracts on financial assets (currencies, interest rates, securities and stock indexes), where the futures price usually corresponds to the spot price, plus the financing costs related to the investment in the underlying asset, minus the remuneration of the underlying asset. For example, the futures price of a contract on fixed income securities should equal the cash market price of the underlying financial instrument, plus the carrying charges associated to this financial instrument (financing costs of the position on the cash market), minus the carry benefits (interest income). Likewise, the futures prices of an equity index should correspond to the financial cash value of the index, plus the financing costs on this index, minus the dividends on

[2] This notation implicitly signifies that there are term structures of convenience yields. This article is however not focused on term structure relationships in commodity markets, the latter being only marginally mentioned in the fourth section. For more information on term structures, see for example Lautier (2005).

the carry period. Therefore, provided some adaptations are made, the storage theory gives the opportunity to apply the contingent claim analysis to commodities, because the convenience yield can be seen as the – implicit – remuneration on the underlying asset of the derivative instrument. This possibility to compare commodities with financial underlying assets partly explains the success of the storage theory, compared with the one of the hedging pressure theory (see box 1).

Box 1. *The hedging pressure theory*

Keynes introduced what he called the "normal backwardation theory" in 1930. In short, this theory specifies that in normal functioning conditions, the commodity market is characterized by a forward price that is inferior to the spot price. The relationship linking these two prices is due to the relative importance of short and long hedging positions in the futures market. Keynes assumes that the market in unbalanced: short hedging positions are inferior to long ones. The presence of the speculators is required in order to restore the equilibrium. Yet, speculators must be rewarded for the risks they experience in their professional activities. Thus, there must be a difference between the forward and the expected spot price: the risk premium;

"...in normal conditions the spot price exceeds the forward price i.e. there is backwardation. In other words, the normal supply price on the spot includes remuneration for the risk of price fluctuation during the period of production, whilst the forward price excludes this."[3]

The critiques raised against the normal backwardation theory stand that even if this premium exists, chances are few that it is positive and constant. The works initiated by Dusak (1972), followed by Bodie and Rosansky (1980), Richard and Sundaresan (1981), as well as Bessembinder (1993) clearly illustrates that phenomenon. So the modern version of the theory only focuses on the hedging unbalance and its effects on the prices relationship. Hence it is called, since the work of de Roon *et al* (2000), the "hedging pressure theory".

The parallel with other financial markets must however be moderated. In commodity markets, negative prices spreads appear when stocks are low (the scarcity can be effective or expected).Thus, reverse cash and carry arbitrages are all the more unlikely that shortage is pronounced: the operators on the physical market do not have any interest in selling the merchandise as long as they expect an additional rise in the spot price. So the basis behaves not in the same way when it is positive and negative. In contango, there are surplus stocks and, as long as storage capacities are not saturated, the basis is stable and limited to the storage costs. In backwardation however, stocks are rare and the basis is solely determined by the spot price the operators are willing to pay in order to immediately obtain the merchandise. There is no subjective limit to the basis. Moreover, because inventories are not sufficiently abundant to absorb the fluctuations in the demand, the spot price is volatile, and so is the basis.

So, difficulties with arbitrage in commodity markets (see box 2) mean that even if the convenience yield restores the non arbitrage relationship between spot and futures prices, this relationship remains specific: the basis has an asymmetrical behavior, due to the non negativity constraint on inventories. The link between present and future exists; however, it is not perfect.

[3] Keynes, 1930.

Box 2. *The difficulties of arbitrage in commodity markets*

In commodity markets, there are at least six sources of imperfections which naturally influence the prices' behaviour.

● Arbitrage can be slowed down by difficulties associated to the localization of the delivery points of the futures contract of the commodity. Indeed, an operator undertaking a cash and carry between the physical and futures markets must necessarily deliver the commodity at the expiration of the contract. However, the wheat used as a support of the futures contract negotiated at the Chicago Board of Trade can be delivered in two places only: Duluth, or Chicago. Yet, these two towns are far from each other and for a buyer, it is not equivalent at all to receive the merchandise in one or the other places. On the other hand, the non ferrous metals underlying the futures contracts negotiated in the London Metal Exchange (LME) can be delivered in more than 60 different places in the world… which gives much more flexibility. Still, delivery is not evident so far. What if a British buyer is proposed to be delivered in Gdansk? Obviously, the question of delivery is not the same for commodities and Treasury Bonds, even when omitting some particular commodities, such as livestock which must be accommodated, cleaned and fed several times a day by the clearing house…

● The storage costs and the necessity to have at one's disposal storage capacities may also hinder arbitrage operations. In this respect, all operators are not created equal. As far as financial assets are concerned, the storage cost does not change from one operator to the other one: it amounts to the financing cost of the position. Things are different in the case of raw materials. For example, the wheat's storage cost is not the same, according as one has at his disposal automated storage capacities or not. The storage cost strongly changes, in the case of petroleum, according as the capacities are located in warm or cold areas. Lastly, all the operators do not necessarily benefit from storage capacities.

● The question of transport is another potential obstacle to arbitrage operations. Once again, all operators do not necessarily possess some transport capacities. Moreover, boats are not necessarily available in the right place and time! A shortage of boats for example, can lead to an immediate and extremely strong backwardation of the metals' futures prices of the LME (in this kind of situation, rises of 40% in the nearest futures price of aluminium have been witnessed). A strike preventing from the exploitation of the unique railway used for the transportation of raw materials can hinder a delivery. And the seller who announces to the clearing house that he can't meet its delivery commitment will probably discover that, as far as the clearing house is concerned, a strike is not a case of overpowering circumstances. Similarly, a strike having an impact on production can have a very important influence on prices… There is no strike in the production of Treasury Bonds.

● Arbitrage can also become particularly because the regarded commodity is not storable. This is the case for electricity. In this situation, in order to benefit from sudden prices rises, one must have production capacities that simultaneously available and flexible. Hydraulic power plants are undoubtedly the most useful tool in this situation. Naturally, the fixed costs associated with such secondary capacities must be sustained when they are not used…

● Merchandises are also not homogeneous in their quality. For example, light crude oils allow to obtain more gasoline than sour crude oils. A possible consequence of such quality differentials is the so-called phenomenon of the "cheapest to deliver". At the time of delivery of a futures contract, the buyer can rationally expect that the seller will try to deliver the lowest quality of the commodity. This problem is very often mentioned and commented upon in the case of futures contracts on interest rates. It has always existed in the context of commodity markets. The trouble is, the problem of the cheapest on rye or oats can not be solved thanks to the calculation of a concordance factor. Thus, it might have an influence on prices.

● Finally, in most of the financial markets, in order to undertake arbitrage operations, it is possible to borrow the assets and to carry out short sales. This is not possible in the case of commodities. Who, in backwardation, would take the risk to leave his merchandise during a few weeks or months and sustain a production's disruption?

2. The motivations for stock holding

Different kind of explanations where provided through time in the literature for the concept of the convenience yield and the motivations explaining storage behavior. We propose to collect and organize them into three categories: the ones that justify stock holding for speculation purposes, those who stress the advantages of inventories as a way to avoid frictions, and those interested in the insurance against stock-out provided by with stock holding.

● *Stocks and speculation*

Speculation is frequently invoked to explain the storage behavior of precious metals. For the other commodities, it is easier to speculate with futures than with physical inventories. This does not mean, however, that stock holding is totally free from speculative motives. As soon as in 1958, while focusing on the convenience yield, Brennan states that stocks give the possibility to benefit from an unexpected rise in the demand without having to wait for supply:

> *"The convenience yield is attributed to the advantage [...] of being able to take advantage of a rise in demand and price without resorting to a revision of the production schedule."*

The speculative motive is also retained by Routledge Seppi and Spatt (2000). They consider that trading profit is the only motive for holding inventory, as far as stocks in excess of those irreversibly committed to production are concerned. In such a situation, stocks can be seen as a way to allocate resources in time. They result from the relation between the present and expected spot prices. This relationship has been pointed out by Deaton and Laroque in 1992. They showed that in equilibrium, the profit maximization of the trader in the physical commodity market implies the following "no arbitrage" conditions:

$$\begin{cases} I(t) = 0 \; if \; \theta E_t\left[S(t+1)\right] < S(t) \\ I(t) \geq 0 \; if \; \theta E_t\left[S(t+1)\right] = S(t) \end{cases} \qquad [3]$$

Where: - $I(t)$ is the quantity stored at t,

- θ is a discounting factor[4]
- $E_t[.]$ is the expectation operator at t,
- $S(t)$ is the spot price at t.

This relationship is interesting because it stresses the physical non negativity constraint on stocks which characterizes commodity markets. However, as long as the speculative motive is retained, nothing is said about stocks held in backwardation.

● *Stocks and frictions*

Nicholas Kaldor was the first to propose the notion of a convenience yield. His definition implicitly relies on frictions in order to explain the storage behaviour when prices' spreads are lower than carrying charges. According to him, when he holds inventory, an operator has the commodity at his disposal as soon as he needs it, and he doesn't have to bear the cost associated to frequent orders. Nor does he have to wait for delivery. This advantage is all the more appreciated that the level of stocks is low:

> *"In normal circumstances, stocks of all goods possess a yield, measured in terms of themselves, and this yield which is a compensation to the holder of stocks, must be deducted from carrying costs proper in calculating net carrying cost. The latter can, therefore, be negative or positive."*[5]

[4] This discounting factor usually stands for financing costs and obsolescence. If r is the interest rate and β stands for obsolescence, then : $\theta = (1-\beta)/(1+r)$.
[5] Kaldor, 1939.

So physical stocks give the possibility to avoid the cost of waiting for delivery; this explains their presence in backwardated markets. The definition proposed by Brennan (1958) relies at least partially on the same argumentation. In 1986, Williams provides another explanation. He claims that, as for money, there is a transaction demand for inventories. The latter indeed give the possibility to undertake transactions immediately; they insure the access to the merchandise. The transaction demand is due to transformation costs, which are much higher for commodities than for money. They stand, first for the costs of buying and selling the commodity, second for processing and transportation expenses. As the example presented in Box n°3 illustrates it, there is a transaction demand even when there is no uncertainty. In the same vein, in 2004, Bobenrieth *et al.* propose to introduce marketing costs in the analysis. According to them, stocks are held neither because they are profitable to keep, nor because they avoid the cost of delivery: they are only expensive to sell.

Box 3. *The transaction demand for inventories (Williams, 1986)*

Let us consider a framework with two periods t_1 and t_2. Just after the harvest, a farmer possesses W bushels of wheat. He can sell them at t_1 and/or at t_2. The amount marketed at t_i, w_i, is such that $w_1 + w_2 = W$. There is no uncertainty: the prices p_1 and p_2 are known. Suppose now that there is backwardation, in that the spot price of wheat is higher than the discounted future price: $p_1 > \partial p_2$, where ∂ is the discounting factor.

In such a simple model, when there are no marketing costs, it is optimal to sell W at t_1. There is no incentive to store the commodity, as suggested by equation [3]. Let us now introduce marketing costs in the analysis, and let us assume that they can be characterized by a non linear function, such as cw^2, where c is a positive constant. With these costs, the optimal storage policy must maximize the present value of the farmer's profit, that is to say:

$$\underset{W=w_1+w_2}{Max}\left(p_1 w_1 - c w_1^2 + \delta\left(p_2 w_2 - c w_2^2\right)\right)$$

The farmer is concerned with finding the optimal amount w_2 that will be stored in the first period and marketed at t_2. Taking the first derivative of the profit function under the constraint that $w_1 + w_2 = W$ gives:

$$w_2^* = \left[\frac{1}{1+\delta}\right]\left[\frac{\delta p_2 - p_1}{2c} + W\right]$$

Thus, even if the spot price is higher than the discounted future price, when transaction costs are introduced in the analysis, positive inventories will be carried from the first to the second period, as long as the following relationship holds:

$$p_1 - \delta p_2 < 2cW$$

Consequently, even if there is no uncertainty on prices, there is still a transaction demand for inventories.

All these explanations share some common points. First, stock holding is explained without relying on uncertainty in the demand and/or in the supply side. Second, the motives of the operators in the physical market are guided by the presence of different kind of frictions. The latter may be external – this is the case for the furnishing delay – as well as internal – as for manipulation or marketing costs.

● *Inventory as an insurance against shortages*

The last category of definitions proposed for the convenience yield considers inventory as an insurance against stock-out. Brennan (1958) stress the advantage of being able to keep regular customers satisfied. Working (1949) relies on the presence of high fixed costs in storage activity and on the necessity to avoid a disruption of activity. Through the concept of stock-out yield, Weymar (1968) also refers to the presence of high fixed costs in production and processing activities. The stock-out yield is linked to the necessity to ensure the continuity of the production. The more important is the risk of a disruption, the more rises the stock-out yield: it is a decreasing function of stocks level. In this context, physical stocks are hold in backwardation because production, storage, transportation and processing capacities are not adjusted to the level of activity: there is overcapacity. Such a situation can be due either to the seasonality of commodities or to a mistake in the forecast of the activity level when capacities were built. As for Williams (1986), he invokes the precautionary demand for inventories as a response, first to the uncertainty that affects the forthcoming supply and/or demand of the commodity, second to the transformation costs (transportation, processing and marketing costs). Box n°4 illustrates this notion.

Box 4. *The precautionary demand for inventories (Williams, 1986)*

In this example, the precautionary demand comes from the rigidity and uncertainty of the production process.

Picture a miller with a fixed production capacity K and a variable cost corresponding to the price of the raw material employed (wheat or corn). This production process is rigid: the firm loses money as soon as it does not operate at full capacity. The miller is concerned with minimizing his expected shortage costs.

In this model, there is also uncertainty. The firm indeed is unable to control the amount of wheat being forwarded to it, and its time to arrival. In order to avoid shortage costs, the miller holds a precautionary stock I.

Let f(z) being the probability that a particular amount of commodity, z, arrives. If this amount is to low to operate at full capacity, the firm will suffer a shortage cost:

$$(K - I - z)\, c,$$

where c is the constant loss from the shortage.

The expected shortage cost of the miller can be written:

$$\int_{0}^{K-I} \left[\left(K - I - z \right) c \right] f(z) dz$$

The problem is how much inventory to keep in hand in order to avoid these costs. In other words, the miller must sole an exercise in inventory control, in which he must find an equilibrium between shortage and storage costs.

The program of the miller is to minimize his total expected costs (EC):

$$\underset{I \geq 0}{MIN} \left(EC \right) = IP_A + \int_{0}^{K-I} \left[\left(K - I - x \right) c \right] f(x) dx$$

In this setting, the secure amount of raw material I stands for a line of credit: it gives immediate access to the commodity. P_A is the price for this service per unit of I; it is also the cost of holding the inventory. The optimum value of I is I* such as:

$$\frac{\delta EC}{\delta I} = 0 = P_A - \int_{0}^{k-I^*} c f(x) dx$$

The optimal inventory is where the price of holding the marginal unit of inventory equals the expected cost of storage.

In this context, two arguments simultaneously explain stock holding in backwardation: the uncertainty affecting demand and supply and the rigidity of industrial and commercial activities including the merchandises. This rigidity finds its expression in the impossibility, for the operators, to quickly adopt their supply to a variation in the demand.

3. Modeling issues

The convenience yield was represented in different ways trough time in the literature. Its simple use as a tool to restore the no-arbitrage relationship gives rise to the conclusion that the convenience yield should be a negative function of the stocks' level. The convenience yield is high when stocks are rare, the holding of stocks being more valuable in this circumstance. Conversely, it is low when there are surplus stocks in the physical markets. Moreover, as stocks and spot prices are also negatively correlated, the convenience yield is a positive function of the spot price. Consequently, the most easy and intuitive way to represent the convenience yield is to regard it as a positive and deterministic function of the spot price, like in Brennan and Schwartz (1985), Gibson and Schwartz (1989) and Brennan (1991).

As soon as in 1989, however, Gibson and Schwartz suggest that the convenience yield may not be perfectly correlated to the spot price and could instead be regarded as a stochastic variable having a mean reverting behavior. This formalization has been chosen by, among others, Gibson and Schwartz (1990), Schwartz (1997), Schwartz (1998) and Cortazar and Schwartz (2003):

$$dC = \left[\kappa\left(\alpha - C\right)\right]dt + \sigma_C dz_C \qquad [4]$$

with: κ, $\sigma_C > 0$

where: - C is the convenience yield,

- α is the long run mean of the convenience yield,
- κ is the speed of adjustment,
- σ_C is the volatility,
- dz_C is an increment to a standard Brownian motion associated with *C*.

Applying a mean reverting behavior to the convenience yield implies that stocks have the capacity to reconstitute themselves, and that there is a level of stocks which satisfies the needs of the industry in normal conditions. The behavior of the operators guarantees that this level is maintained. When the convenience yield is low, stocks are abundant and they bear a high storage cost reckoning the poor benefits associated with the holding of the merchandise. Consequently, if they are rational they will try to reduce this surplus inventories. Conversely, in the case of a shortage, the operators will try to reconstitute their stocks.

The formalization proposed by Schwartz (1997) is especially interesting: not only does he retain a mean reverting behavior for the convenience yield, but he also introduces this variable in the spot price's dynamic. Thus – marginally – the former gives to the latter a mean reverting behavior. More importantly, the convenience yield becomes a stochastic dividend influencing the drift of the spot price. This representation states explicitly the convenience yield as a profit associated with the holding of inventories and facilitates the comparison with other financial assets, such as stocks or bonds.

Meanwhile, the formalization gained in complexity, as the authors began to take into account the non-negativity constraint on inventory. These works were initiated by Brennan, in 1991. The author claims that the convenience yield has an asymmetrical behavior and is downward limited. More precisely, the convenience yield net of storage costs can not be lower than the opposite of storage costs. The latter are supposed constant for a large range of prices, as long as the storage capacities are not saturated. In their model, Routledge, Seppi and Spatt (2000) suppose that the correlation between the spot price and the convenience yield is higher in backwardation than in contango. While being stochastic, the convenience yield is an endogenous variable which is caused by the storage process. Lastly, Lautier and Galli (2001) propose a generalized version of Schwartz' model (1997), where the convenience yield is mean reverting but is also higher in backwardation than in contango.

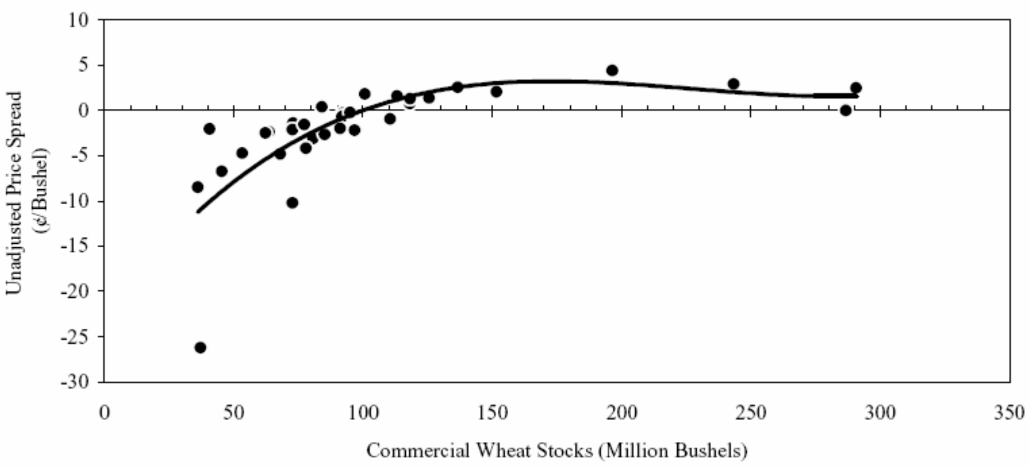
An option being, above all, an asymmetrical asset, all these works can be considered as a first step towards the representation of the convenience yield as a real option. In this context, the convenience yield is represented as a call option on inventory. According to Heinkel, Howes, and Hughes (1990), the option lies in the possibility offered by the inventory to profit from an unexpected rise in the demand. In their model, the convenience yield is a decreasing function of the stock level and an increasing function of production costs. In Milonas and Henker (2001), the value of the call is zero if there is no disruption in the production cycle. When the stock rises, the probability of a disruption diminishes, as well as the option's value. Likewise, the more the price of the underlying asset is volatile, the more the option value increases. The theoretical framework is however restricted to a single production cycle, beginning when stocks are bought and finishing when they are entirely sold. It is well suited for seasonal commodities like agricultural products, but more questionable for industrial raw materials such as metals, where production cycles do not have a strong influence on prices in the short run. Relying on Longstaff (1995), Heaney (2002) proposes an approximation of the real option's value associated with the convenience yield, where the trader holding the commodity is assumed to know when the price reaches its highest value over the period of storage. This model provides a maximum value for the call option.

4. Empirical evidence

The concept of convenience yield has given rise to two different kind of empirical tests. The first directly rely on stock data in order to give evidence of a convenience yield in commodity markets. Recognizing the difficulties in obtaining reliable stock data, the second preferably rely on price data and perform indirect tests. Even if this second category is interesting, our aim being to see whether or not the convenience yield is something else than an *ad-hoc* "correction" of the prices relationship in commodity markets, we will focus on the first. As far as the second category is concerned, let us only point out that the different models mentioned above were most of the time empirically validated with quite a large consensus.

If there is a convenience yield related to physical ownership, then the holding of inventory in backwardated markets becomes rational. Thus a first step towards the justification of the concept lies in empirical observations linking the stocks level and prices spreads. As soon as in 1933, Working presented a stylized "supply of storage curve" which, as illustrated by Figure 2, is a non linear and negative relationship between inventory and prices spreads. Noticeably, on the wheat market, firms do store commodities at a loss: when prices spreads become negative, the stock level remains around 50 millions of bushels.

Figure 2. Working curve for wheat: US commercial stocks



Source: Working (1933).
Note: The data used in the figure are the unadjusted price spread, i.e., the September–July wheat price spread and the July 1st total U.S. commercial stocks 1896–1932.

Source : Carter & Giha (2007)

Empirical estimates of the supply of storage generally confirmed this result, on an important number of commodity markets, as illustrated by Table 1.

Table 1. Empirical evidence of the Working curve through time and commodity markets: a few examples

Author(s)	Commodity market(s)	Date
Brennan	Shell eggs, cheese, butter, oats	1958
Telser	Cotton	1958
Weymar	Cocoa	1966
Gray & Peck	Wheat	1981
Thompson	Coffee	1986
Williams and Whright	Coffee	1989
Cho & Mc Dougall	Crude oil and petroleum products	1990
Brennan, Williams & Wright	Wheat	1997
Carter & Giha	Wheat	2007
Lin & Duan	Crude oil	2007

This negative relationship between stocks and prices may be considered as an evidence of the existence of a convenience yield. With time, the analysis of this empirical phenomenon became more and more precise. As soon as 1958, Telser suggests that the convenience yield associated with strategic government stocks is lower than the convenience yield of industrial stocks. Thompson (1986) shows that the price-stock relationship can be stronger when local stocks are taken into account, instead of worldwide stocks. Williams and Wright (1989) confirm this result with stocks certified by the exchange. Indeed, a difficulty arising when considering inventory is that there are several kind of stocks: on can consider physical versus paper inventories; speculative and/or industrial stocks, strategic and/or non strategic stocks, stocks that are underground (for mineral reserves) or not, stocks certified by the clearing house or not, etc. Obviously, the results of the empirical tests change with the category which is taken into account. This remark constitutes the basis of the main critique addressed to the convenience yield.

5. Critiques

The absence of direct evidence of convenience yield gives rises to critiques. Two categories of objections were raised. The first is associated to the non observable nature of convenience yield. Indeed, there is no real traded asset corresponding to this variable. This objection is however irrelevant: in the field of finance any expected variable is non observable. It is the case, for example, for futures spot prices or for expected inflation.

The second category of objection is more serious. It was initiated by Williams and Wright (1989). They claim that the convenience yield is an artifact of an aggregation problem. Stocks indeed may group together, in homogeneous statistical data, some different commodities, which are not really substitutable, because they are characterized by quality differential, and / or because they are situated in different locations, that are far away from each others. In such a situation, storage under backwardation may occur, not because there is an implicit remuneration associated to inventories, but because aggregated stock data give a false image of the price-inventory relationship.

A possible answer to such a critique is that when a very restrictive kind of stock – namely those certified by the clearing house – is retained in order to analyze the relationship between stocks and prices, there is still evidence of positive stocks in backwardation. Thus, even if the convenience yield is probably often overestimated, it is impossible to deny that firms store stocks at an apparent loss in commodity markets. Consequently, whatever the name we put on it, the physical ownership of the commodity obviously carries a flow of services.

6. What about non storable commodities?

The emergence and development of derivative markets for "non storable" commodities, like electricity or carbon, challenge the concept of the convenience yield, which was initially proposed in the context of storable commodities. Is it possible to keep it in this new context? What will it stand for, in this situation? The answer given in this article is tentative, as the subject and the markets are recent[6].

As far as electricity is concerned, only a small part of the total production of electricity (namely hydroelectricity) is storable. Thus, is the concept of convenience yield irrelevant for power markets? Géman (2005) claims that the convenience yield cannot be extended to electricity. The latter being a non storable commodity, there is no possibility of carrying it over time. However, she considers the convenience yield as it is most currently defined. Provided that a larger definition is retained, the concept can remain useful. As is the case for most commodities production process, power plants indeed require investing very high amount of capital. Moreover, the so called committed generation units[7], kept as a reserve, have a role in power markets which is comparable to that of inventories.

The context of the carbon market being quite different depending on the geographical area is concerned; this article will focus on the European one. This market on emission rights was launched in two phases: 2005-2007 and 2008-2012. The carbon market is a bit specific as until now, it is not possible to store the underlying asset of the futures contracts, namely the carbon – even if the CSC (Carbone Storage and Capture) technology is quickly improving. It is however possible to store the emission rights, at least, since 2008[8]. Others characteristics of the carbon market are first, that the pure storage cost of CO2 allowances is zero, and second that their supply is punctual. Consequently, if as claimed by Trück *et al* (2007) or Daskalis *et al* (2009), there is a convenience yield associated to the permits, it is probably only due to the scarcity of CO_2 allowances, which might be either observed or expected. Moreover, the form of the supply function of the emission rights should induce a cyclical

[6] It is generally accepted that a derivative market is mature when it is 10 to 15 years old. The immaturity of the market renders the analysis of prices behavior spurious.
[7] "A generation unit is said to be committed if it can be turned on, brought up to the desirable speed and connected to the system in order to deliver power to the network, all these steps taking place in a very short amount of time" (Géman, 2005).
[8] The first period of the European market, from 2005 to 2007, was a trial period characterized by a banking prohibition: it was impossible to save the permits from the first to the second phase. Thus, it has exhibited very specific prices' behavior. Uhrig-Hombourg and Wagner (2008) claim that the contango observed during the first period is due to carbon markets' immaturity and does not extend to the second period. The banking restriction prevents the cost-of-carry approach from working across the different phases. Daskalakis *et al* (2009) also mention that the banking mechanism has a significant effect on futures prices.

behavior for the convenience yield. Trück *et al* (2007) call it a seasonal behavior, whereas Daskalis et al (2009) refer to mean reversion.

7. Conclusion

This article explains the role of the convenience yield in the relationship between spot and futures prices in commodity derivatives markets. This variable is essential for several reasons. First, it is a tool that explains backwardation in commodity markets. Second, it allows establishing conceptual and methodological links between commodity derivatives markets and other assets, like bonds, stocks, and currencies. Such links can be very useful, as illustrated by the success of the analogy between financial and real options. Third, it explains why firms keep storing commodities at negative prices spreads.

The convenience is however a controversial concept. Indeed, even if its first formulations go back to the beginning of the twentieth century, there is no real consensus on its definition. The one deliberately retained in this article is as large as possible. It aims at explaining backwardation in commodity markets and gathering the reasonable explanations, having an economical meaning, that were proposed trough time in the literature. In such a context, it is not limited to physical inventories and it can be exploited for "non storable" commodities such as electricity or carbon.

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