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**Natural and Industrial Disasters: Land Use and Insurance**

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# Natural and Industrial Disasters: Land Use and Insurance\*

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## Abstract

Uncontrolled urbanization in exposed areas increases the cost of natural and industrial disasters. In the case of industrial risks, these land use externalities are directly exerted on the firm which is liable for the disaster. In the case of natural risks, the externalities take the form of free-riding on the mutualization mechanism. Land use policy and insurance can provide incentives to internalize these externalities. However these tools cannot be more precise than the hazard map on which they are based. We describe the efficiency gains of refining hazard categories, keeping in mind that they should be balanced with assessment costs. We determine the impact of climate change and demographic evolution on the equilibrium. Our results are established for a spectrum of alternative scenarios with various distributions of bargaining power between the households, the mayor and the firm.

*Keywords:* natural disasters, industrial disasters, insurance, land use regulation, hazard maps

*JEL classification:* H23, G22, R52, Q54

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# 1 Introduction

Many areas exposed to catastrophic risks are inhabited and used for economic activities. According to Zhang (2004), in China, “about 8% of the land area located in the mid- and down stream parts of the seven major rivers of the country [is] prone to floods”. However, 50% of the country’s total population lives in these areas, and “they contribute over 2/3 of total agricultural and industrial product value”. According to Doherty et al. (2008), “in 2004, [...] the modeling firm AIR Worldwide estimated that nearly 80 percent of insured assets in Florida are located near the coasts, the high-risk area in the state. This represents \$1.9 trillion of insured exposure located in coastal areas (commercial and residential exposure)”.

The magnitude of the disaster which occurred in 1984 at the Union Carbide India Limited pesticide plant in Bhopal (India) is partly explained by uncontrolled urbanization in the vicinity of the plant. “The population of Bhopal stood at 102,000 in 1966. After Union Carbide and other industries settled there in the 1960s, the population grew to 385,000 in 1971, 670,000 in 1981, and 800,000 in 1984” (Ferrante (2011)). Creeping urbanization is exemplified by neighborhoods near to the AZF plant in Toulouse (France), which exploded in 2001. The AZF website mentions that “initially [in 1924] far from the dwelling areas, the plant has been progressively bordered by the Toulouse agglomeration”; pictures of the plant and its neighborhoods in the 1930s and in 2001 illustrate the agglomeration’s expansion.<sup>1</sup>

Indeed, populations are drawn to risky locations by associated amenities such as river views, by jobs in industrial areas, or because they are fleeing expensive centers. The intertwined histories of risk and urban development can be complex but the law is quite simple: there is no right of “initial land use” by which the community or the industrialist can renege on its responsibility for any disaster compensation to the newcomers.<sup>2</sup>

The cost of natural and industrial risks is strongly determined by the number of people and businesses located in exposed areas and the value of their assets. “Changes in population, inflation and per capita real wealth are the main factors contributing to the increase of the original raw losses” due to floods in Europe between 1970 and 2006; “[a]fter filtering their influence there remains no evident signal suggesting any influence of anthropogenic climate change on the trend of flood losses in Europe during the assessed period” (Barredo (2009)). Similarly, “insured losses from earthquakes are rising because population growth and higher population density, especially in urban areas, expose more people to a single damaging earthquake [...] the seismic threat itself remains unchanged” (Bevere et al. (2011)).

White and Haas (1975) deplore that research on natural disasters has mainly focused on physics and engineering while overlooking the economic, social and political dimensions of these events. Mileti (1999), in his book *Disasters by Design*, confirms the underinvestment in these important fields but observes that in the 1990s policies have evolved in this respect. Risk is the combination of hazard, i.e. the probability that a loss occurs, and vulnerability, i.e. the value that could be lost. Here, we leave aside hazard reduction (dams, levees or safety-oriented industrial design) and building standards (antiseismic systems or improved air tightness in protection

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<sup>1</sup>See <http://www.azf.fr/1-usine-azf-de-toulouse/historique-800233.html>.

<sup>2</sup>For industrial disasters in France, see the Environment Code (section L 514-19), the rules of civil liability (Civil Code, section 1382 and the following ones) and legal precedents. A unique exception can be invoked: constant and chronic pollution that would induce nuisances but neither material nor health damages (Code of Building and Living, section L 112-16).

against toxic fumes). We assume that all technical risk reduction measures have already been taken. As the topic has been neglected, our analysis focuses on vulnerability, inasmuch as it reflects urbanization choices at the individual and collective levels.

Land use externalities, namely the fact that urbanization in risky localities increases the cost of potential disasters, are the focus of this paper. In the case of industrial risks, the externality is directly exerted on the firm. In the case of natural risks, the externality takes the form of free-riding when the mutualization of risk (via insurance or other solidarity mechanisms, namely tax-funded aid) provides no incentive to households to locate efficiently. [Morgan \(2007\)](#) explains that in Santa Rosa County (Florida), “subsidized insurance premia create a [real estate] market imbalance by reducing expected flood losses [...] in floodplain areas”. Clearly, several vulnerability decreasing policies based on land use or insurance regulations can internalize land use externalities: density restrictions, adjusted insurance premiums, and location-dependent taxes all contribute to shaping equilibrium risk exposure. We provide examples of these policies.

The community or industrialists can limit urbanization in exposed areas. The State can appropriate land, prohibit new building or limit population density to reduce exposure to natural risks. After the Great Flood of 1993 in the United States, “across nine states in the Midwest, more than 9,000 homeowners sold their properties” ([FEMA \(2008\)](#)); “entire towns, such as Valmeyer, Illinois, decided to move from the floodplain to higher ground, breaking an ongoing cycle of flood damage and government relief spending” ([Bagstad et al. \(2007\)](#)). Similarly, an industrial firm can purchase or rent land, establishing a red zone in the vicinity of its plants. In the United States, “for example, in 1991 the company Dow Chemical paid to relocate a village of 300 inhabitants, which was situated very close to one of its chemical plants in Louisiana” ([Sauvage \(1997\)](#), our translation). In practice, when land is purchased by the firm, the conditions are negotiated between the firm and the mayor.

Insurance premiums for households and businesses can also provide incentives to locate in less exposed areas and therefore internalize (at least partly) land use externalities. Empirical works confirm that insurance tariffs are reflected in housing markets, real estate prices and density. Real estate prices respond even more to insurance premiums than to some other risk revelations: in Houston (Texas) in the 1980s, real estate prices did not immediately decline after a flood, but “when flood insurance premiums r[ose] dramatically approximately one year after the [1979] flood, these higher rates [were] capitalized into home values and prices [did] decline” ([Skantz and Strickland \(1987\)](#)). As the cost of industrial disasters is borne by firms, households and businesses do not need to purchase insurance against these risks, except perhaps via legal assistance insurance.<sup>3</sup> However, location-dependent taxes (or equivalently obligations with differentiated financial incidence) can play an incentive role similar to that played by insurance. For example, after the 2001 AZF accident in France, a series of costly construction works were imposed in the vicinity of hazardous plants. The extent to which the community is compensated by the firm for this extra burden is the most relevant political issue.

In this paper, we analyze households’ equilibrium location choices and risk exposure given the insurance tariffs and organization of the real estate market. When households do not pay fully for the risk they take, they exert land use externalities on society; these are key to our analysis. For natural disasters, free-riding is an unintended consequence of solidarity mechanisms.

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<sup>3</sup>For example, the aim of the French insurance system is only to manage the basic coverage for victims by avoiding long litigation and by covering the residual risk of no responsible identification. In many other countries, households directly lodge a complaint against the firm and no such insurance system is organized.

For industrial disasters, ex post compensation by the industrialist hardly encourages preventive measures from households. Our approach leaves aside the issue of imperfect compensation (delays, defaults, deductibles)<sup>4</sup> and does not consider cognitive biases on risk perception.<sup>5</sup> This is not to deny the practical importance of these phenomena, but rather to propose a basic and pure analysis of the common types of real world institutions.

[Picard \(2008\)](#) proposes a model in which households' locations are fixed but differ with respect to their exposure to natural disasters. Actuarial insurance would be efficient, since it would induce consumers to invest optimally in prevention and mitigation. However, inequalities would be inevitably attached to the individualized tariffs. For that reason, transfers could be promoted. Here, we focus instead on competitive location choices and thus adopt the long-term perspective.

In a model derived from classical urban economics literature (see [Fujita and Thisse \(2002\)](#) for a review), [Frame \(1998\)](#) takes into account a second spatial dimension: locations not only vary in terms of distance to the center, but also in terms of risk. The paper proposes comparative statics on the equilibrium variables for each of two cases: where households have to absorb losses by themselves and where actuarial insurance (with loadings) is available. We focus instead on the degree of insurance price discrimination between locations and its equilibrium consequences.

[Frame \(2001\)](#) shows that starting from a situation without insurance or with imperfect insurance (for example because of loadings), a small dose of uniform coverage increases welfare. The intuition is that making exposed areas less daunting benefits everyone via an alleviation of urban congestion. In fact, his result is only local: complete uniform insurance would unleash land use externalities and may not Pareto dominate the absence of insurance. Our objective is precisely to analyze the complementary policies that counteract these undesirable effects.

The first contribution of our paper is to analyze hazard maps on which land use and insurance regulations are based. Regulators and insurers use maps delineating location-based risk segments to differentiate land use restrictions and insurance tariffs. The precision of the maps depends on scientific and technical limitations, as it requires historical data, engineering and prospective analysis (climate change for example, or reliability analysis for industry). Unsurprisingly, refining the map provides efficiency gains that should be balanced with risk assessment costs. It also triggers political disputes, since a more accurate map may open the way to more discriminating land use or insurance regulations.

The second contribution concerns the impact of the parameters on the equilibrium policy. Although currently the increasing cost of natural disasters is largely explained by the growing urbanization of risky areas, in the future climate change could have a proportionately greater impact. We examine both factors. In most scenarios, an increasing frequency of disasters justifies extension of the exclusion zone (red zone in this paper). The magnitude of the response varies between the scenarios in ways that are instructive. Population increase has, by nature, ambiguous consequences. It increases not only the number of potential victims but also the number of people looking for space to live in. The notion of sanctuaries summarizes interesting balances between the conflicting effects.

The third contribution of our paper is to offer a spectrum of alternative scenarios. We analyze

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<sup>4</sup>In practice, imperfect compensation for disasters implies that a part of their cost is borne by households. This reduces the importance of land use externalities.

<sup>5</sup>[Tatano et al. \(2004\)](#) show that imperfect risk perception should be corrected to enhance market efficiency.

the similarities and differences between the management of natural and industrial risks. We also consider different distributions of bargaining power between households, the mayor (representing the collective interest of households) and the firm (when there is one). All these elements help test the robustness of our results.

Section 2 sets up the model. Section 3 presents the case of natural disasters. After the treatment of the polar examples (actuarial insurance and uniform insurance with a red zone), the notion of hazard maps and their refinement is explored. Section 4 deals with industrial disasters. Several types of organization of real estate markets are studied: scenarios where bargaining power lies with the firm, with the mayor, and more competitive forms. Section 5 draws comparative statics with respect to risk and between scenarios. Section 6 concludes.

## 2 The model

Notations defined here are also listed in Appendix A.1.

**Space and risk.**  $[0; \bar{x}]$  is the set of inhabitable locations (Figure 1). The risk source (e.g. the river bed or the plant) is located at 0. The distance  $x$  between the source and a location determines risk exposure. The safest place  $\bar{x}$  can be seen as a crest.

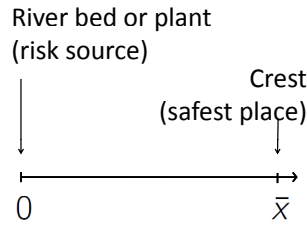


Figure 1: Space and risk

A household lives at a given location, say  $x$ , in  $[0; \bar{x}]$ ; its dwelling occupies a surface  $s(x)$  and the density of households at location  $x$  is  $n(x)$ .  $[0; \bar{x}]$  may have uninhabited intervals. We have

$$\int_0^{\bar{x}} n(x) dx = N, \quad (\text{POP})$$

where  $N$  is the total population (POP). There is surface limitation at every location:

$$\forall x, n(x) s(x) \leq 1.$$

A dwelling located at  $x$  will be damaged only once with probability  $p(x)$  with

$$\forall x, p(x) = \rho f(x), \quad (1)$$

where function  $f(\cdot)$  is positive, decreasing along the space line and piecewise continuous.  $\rho > 0$  is a magnitude index that will be used for comparative statics. The damage per dwelling has two parts, one fixed  $\lambda_F \geq 0$  and the other proportional to the housing surface  $\lambda_S s$  with  $\lambda_S \geq 0$ .

The damage corresponds to (re)building cost and does not depend on land value. There is no damage in empty places.

Risk is a combination of hazard and vulnerability. Here, hazard is measured by  $p(\cdot)$  and vulnerability associated with building techniques is measured by  $\lambda_F$  and  $\lambda_S$ . We assume that all technical risk reduction measures have already been taken. *Endogenous* vulnerability due to location choices (household density and housing surface) is the main focus of the study.

**Insurance.** Within a given community, risks are by nature highly correlated. The large number of communities on a much larger scale makes global risk tolerance high, so that we can assume risk neutrality of the insurance sector. We also assume that insurers have sufficient reserves to absorb any loss so that there is no risk for households not to be fully compensated.

Insurance policies offered to cover a dwelling of surface  $s$  located at  $x$  consist of a premium  $\pi(x, s)$  and a complete reimbursement  $\lambda_F + \lambda_S s$ . Delays and associated costs are supposed to be fully compensated. Therefore risk influences location choices only through  $\pi(x, s)$ . We consider premium functions (PF) with two components, one fixed and the other proportional to surface:

$$\pi(x, s) = \pi_F(x) + \pi_S(x) s. \quad (\text{PF})$$

This conforms well with the structure of risk and it allows us to study and compare actuarial and uniform premiums.

In all scenarios, all households are insured: voluntarily for natural disasters when insurance is actuarial, compulsorily when it is uniform (to avoid adverse selection), and by a third party for industrial disasters.

We assume that natural disasters (ND) compensation takes the form of a pure insurance system without administrative costs:

$$\int_0^{\bar{x}} \pi(x, s(x)) n(x) dx = \int_0^{\bar{x}} p(x) (\lambda_F + \lambda_S s(x)) n(x) dx. \quad (\text{ND})$$

It could be implemented by a perfectly competitive private sector or by an efficient administration.

For industrial disasters (ID), the firm is liable for all damages. There is no need for households to take out insurance:

$$\forall x, \forall s, \pi(x, s) = 0. \quad (\text{ID})$$

The firm itself is assumed to be risk-neutral or equivalently fully insured by risk-neutral insurers.

**Rent.** Buying or perpetually renting a surface is equivalent; so we speak of rents henceforth. The rent  $r(x)$  is the price per unit of surface at  $x$ . The opportunity cost of land is assumed to be nil so that rent is nil in empty places. If  $r(x) = 0$ , supply is in  $[0; 1]$ ; if  $r(x) > 0$ , supply is 1. In equilibrium demand equals supply in each location.

$$n(x)s(x) = \begin{cases} 1 & \text{in inhabited locations,} \\ 0 & \text{otherwise.} \end{cases} \quad (\text{LOC})$$

We assume that the whole land area is owned by a fund of which households have equal shares; this structure makes sure that all reforms have a homogenous effect on households.

Households each receive

$$\bar{r} = \frac{1}{N} \int_0^{\bar{x}} r(x)s(x)n(x)dx, \quad (2)$$

which they take as given. The income effects are worth analyzing; this is why we do not make the “absentee landlord” hypothesis, often encountered in the literature. If one wants to neutralize the income effects, one can take a quasilinear utility function as a special case, as the flexibility of our model allows (see next paragraph).

**Households.** We assume that households are identical and have no intrinsic preference for one location over another. Their utility  $U(z, s)$  depends on their consumption  $z$  of the composite good (henceforth money) and on their housing surface  $s$ .  $U$  is twice differentiable, strictly increasing and strictly quasiconcave with respect to  $z$  and  $s$ . Households are price takers; they have an income  $\omega$  and they maximize their expected utility under their budget constraint. Since insurance is complete, the expected utility  $EU$  is no more than the utility  $U$ . Households’ (HH) choice  $(x, z, s)$  is optimal if it solves

$$\max_{x,z,s} U(z, s) \quad \text{s.t.} \quad \omega + \bar{r} \geq z + sr(x) + \pi(x, s). \quad (\text{HH})$$

We say that  $x$  is inhabited if it is part of a solution of (HH). In the following,  $(z(x), s(x))$  denotes the optimal consumption associated with location  $x$  (inhabited or not).

We denote  $MRS_{sz}$  the marginal rate of substitution of  $s$  with respect to  $z$ , that is

$$MRS_{sz} = \frac{\partial U / \partial s}{\partial U / \partial z}. \quad (3)$$

We use a similar convention for  $MRS_{zs}$ .

**The equilibrium.** As households are free to move, equilibrium is defined by the fact that no household has an incentive to change its location or its housing surface and that local and general constraints are satisfied.

**Definition 1** (Equilibrium). *Let  $\pi(\cdot, \cdot)$  be an insurance pricing allowed by (PF). An equilibrium is a set of functions  $n(\cdot), z(\cdot), s(\cdot), r(\cdot)$  such that the following four conditions are satisfied: (i) (POP), (ii) (ND) for natural disasters or (ID) for industrial disasters, (iii) (HH) and (iv) (LOC).*

In equilibrium, population density, housing surface, location choice and therefore individual and total risk exposures are endogenous and depend on the insurance pricing (e.g. actuarial or uniform). Appendix A.2 determines useful necessary conditions characterizing the equilibrium. The existence of a solution and its essential uniqueness will be proved in the scenarios we will now study.

### 3 Natural disasters

Regulators and insurers use maps delineating location-based risk segments to differentiate land use restrictions and insurance tariffs. In France, natural disasters insurance is based on uniform



premiums;<sup>6</sup> in Japan, the Probabilistic Seismic Hazard Maps delineate the four earthquake premium zones (Tsubokawa (2004)); in the United States the Flood Insurance Rate Maps delimit the flood premium zones (Hayes and Neal (2009)) and flood insurance is actuarial with respect to these maps.

Defining the precision of hazard maps is a political choice, since a more accurate map opens the way to more discriminating land use or insurance regulations. But it also depends on scientific and technical limitations, as it requires historical data, engineering and prospective analysis (climate change for example, or reliability analysis for industry). The precision of hazard maps varies between hazard types. The maps of seismic activity in France exhibit five zones.<sup>7</sup> The ones relative to shrinking and swelling of clay soils are more precise but not reliable at the dwelling scale.<sup>8</sup> Flood maps in the United States are reliable at the street scale.<sup>9</sup> The precision and even the availability of hazard maps also vary from one area to the next due to a lack of standardization. The precision of floods maps in France depends on the implementation stage of a natural risk prevention plan;<sup>10</sup> flood maps in the United States are not provided in all jurisdictions.

Maps shape the real estate market and insurance and have a durable effect on total risk exposure. We first analyze two polar examples: actuarial insurance and uniform insurance with a red zone, i.e., an area where settlement is prohibited. Actuarial insurance corresponds to pricing in compliance with the hazard map reflecting the hazard function  $p(\cdot)$ . On the contrary, uniform insurance with a red zone relies on the simplest maps with two zones: one where land use is prohibited and one where a single insurance tariff is applied. We then describe the efficiency gains of refining hazard categories, keeping in mind that they should be balanced with assessment costs.

### 3.1 Actuarial insurance

With actuarially fair pricing, all households purchase insurance. When households choose location  $x$  and surface  $s(x)$ , they consider the “total” rent  $r(x) + \lambda_S p(x)$  and the fixed part  $\lambda_F p(x)$ . Two locations are equally attractive only if the total rent is smaller where the fixed part is higher. In riskier areas, the fixed part is clearly more expensive, therefore the total rent is necessarily smaller (Equation (22) in Appendix A.2).<sup>11</sup> Along the isoutility curve where all optimal choices are located, smaller total rent is necessarily associated with more demand for space: compensated demand increases as the price decreases. Consequently, in riskier areas, households demand more space and are thus more dispersed. Therefore  $n(\cdot)$  and  $z(\cdot)$  increase with respect to  $x$ , while  $s(\cdot)$  decreases. At the limit, the riskiest areas are deserted. As  $r(\cdot)$  is increasing, either

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<sup>6</sup>The guarantee against natural disasters is mandatorily included in home, business building or car insurance policies. The natural disasters premium is a fixed share of the premium for other damages covered by the insurance policy.

<sup>7</sup>See <http://www.planseisme.fr/spip.php?article19>.

<sup>8</sup>See <http://www.argiles.fr/>.

<sup>9</sup>See <http://www.esri.com/hazards/>.

<sup>10</sup>See <http://cartorisque.prim.net/>.

<sup>11</sup>Several empirical studies based on the hedonic prices method confirm that housing markets value the capitalized flow of natural risks insurance premiums (Bin et al. (2008), MacDonald et al. (1990), Harrison et al. (2001)).

$r(0) = 0$  and we denote  $x^*$  the highest location defined by

$$r(x^*) = 0, \quad (4)$$

or  $r(0) > 0$  and the whole space is inhabited and then  $x^* = 0$ . Therefore  $x^*$  is the leftmost (i.e. riskiest) inhabited location in equilibrium.

**Proposition 1.** *Actuarial insurance pricing implements a Pareto optimum.*

*Proof.* See Appendix A.3. Note that this proof only uses  $p(\cdot)$  decreasing. □

Indeed, actuarial insurance makes households pay the price of risk. Similar effects can be produced with location-dependent limitation of population density instead of location-dependent insurance pricing. Density limitation can be implemented via auctions of occupancy rights.

### 3.2 Red zone under uniform pricing

We will now consider the polar case of uniform insurance. Uniform insurance opens the way to adverse selection and difficulties of mutualization on a large scale. In France and in Spain, uniform insurance is organized by the government and comes automatically with basic property insurance policies (Dumas et al. (2005)). The notion of uniform insurance should not be taken too literally: State assistance funded by taxes has equivalent effects, taxes being similar to insurance premiums.<sup>12</sup> In many countries the insurance market is not developed and the State organizes public aid after disasters (Dumas et al. (2005)). We consider here that all households benefit from compensation after natural disasters at a uniform price.

Clearly, uniform insurance exacerbates land use externalities: households do not pay for the risk they generate by locating in exposed areas. All permitted locations have the same value for households and the building zone is fully and uniformly used. Combining uniform insurance with building restrictions partially corrects imperfect internalization of risk and increases efficiency. The State understands this and prohibits the most exposed areas, thus defining a block that we will henceforth call a *red zone*. In equilibrium,  $x^*$  denotes both the size of the red zone and the leftmost inhabited location (Figure 2). The red zone is the only variable that policy can change to reach an optimum: increasing  $x^*$  reduces the cost of risk and available space at the same time.

When land use is uniform over the inhabited area  $[x^*, \bar{x}]$ , the total expected cost of risk CR amounts to

$$CR(x^*) = \left( \frac{N\lambda_F}{\bar{x} - x^*} + \lambda_S \right) \int_{x^*}^{\bar{x}} p(t) dt. \quad (5)$$

Remark that increasing  $x^*$  decreases the cost of risk (people occupy less risky zones): it leads to a positive marginal risk reduction MRR.

$$MRR(x^*) = -\frac{dCR}{dx} \geq 0. \quad (6)$$

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<sup>12</sup>Coate (1995) claims with reason that the equivalence is less than perfect. Firstly, ex post assistance by the State is less efficient because there is no reason to expect that people who provide assistance will choose the optimal level of assistance: assistance may rely on approximate loss assessments or discretionary decisions. Secondly, natural disasters assistance is provided by various actors (non-profit organizations, States); the uninsured can free-ride. In this respect, the assistance providers themselves can consider that the level of assistance is not optimal. We leave these issues aside.

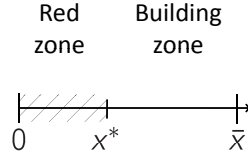


Figure 2: Red zone

We assume that the marginal benefit of reducing the cost of risk decreases.<sup>13</sup>

$$\forall x, \frac{dMRR}{dx} \leq 0. \quad (\text{CVX})$$

**Definition 2** (Constrained optimality). *A red zone is said to be constrained optimal if it is Pareto optimal under the constraint that authorized land is uniformly used by households.*

The constrained-optimal red zone is the solution of the program maximizing utility under the budget constraint:

$$\begin{cases} \max_{(z,x)} U\left(z, \frac{\bar{x}-x}{N}\right), \\ \text{s.t.} \quad \omega + \bar{r} \geq z + \frac{\bar{x}-x}{N}r(x) + \frac{\text{CR}(x)}{N}, \\ 0 \leq x \leq \bar{x}. \end{cases} \Leftrightarrow \begin{cases} \max_{(T,x)} U\left(\omega + \frac{T}{N}, \frac{\bar{x}-x}{N}\right), \\ \text{s.t.} \quad -T \geq \text{CR}(x), \\ 0 \leq x \leq \bar{x}. \end{cases} \quad (7)$$

The uniform premium shares the total expected cost of risk equally between households.

**Proposition 2.** *The following two conditions are equivalent.*

(A)  $x^*$  is the constrained-optimal red zone.

(B)  $x^*$  is the unique solution of

$$\begin{cases} x^* = 0 & \text{and } MRR(0) \leq MRS_{sz}\left(\omega - \frac{\text{CR}(0)}{N}, \frac{\bar{x}}{N}\right), \\ x^* \in (0, \bar{x}) & \text{and } MRR(x^*) = MRS_{sz}\left(\omega - \frac{\text{CR}(x^*)}{N}, \frac{\bar{x}-x^*}{N}\right), \\ x^* = \bar{x} & \text{and } MRR(\bar{x}) \geq MRS_{sz}\left(\omega - \frac{\text{CR}(\bar{x})}{N}, 0\right), \end{cases} \quad (8)$$

where  $MRR$  is the marginal risk reduction and  $MRS_{sz}$  the marginal rate of substitution of households.

*Proof.* The constraint in (7) is convex in  $T$  and in  $x$  (it is linear in  $T$  and  $\text{CR}(\cdot)$  is convex (CVX)). As additionally the objective is strictly quasiconcave, the Kuhn-Tucker conditions can be easily rearranged to give the necessary and sufficient condition (B) defining the unique constrained optimum (A). Note that corner solutions are not excluded.  $\square$

<sup>13</sup>In other words, the cost of risk  $\text{CR}(\cdot)$  is convex. Note that  $p(\cdot)$  convex is a sufficient condition to get  $\text{CR}(\cdot)$  convex.

We denote by  $x_{\text{Nat}}^*$  the constrained-optimal red zone. The utility of all is  $U\left(\omega - \frac{\text{CR}(x_{\text{Nat}}^*)}{N}, \frac{\bar{x} - x_{\text{Nat}}^*}{N}\right)$ . The utility increases with respect to the income  $\omega$ , and decreases with respect to the loss parameters  $\lambda_F$ ,  $\lambda_S$  and  $\rho$ . The impact of an increase of population  $N$  on the utility is ambiguous. Indeed, increasing the number of households reduces the surface occupied by each of them but also dilutes the land use externalities.

In France, where insurance is uniform, the 1995 law created natural risk prevention plans that define red zones where new building is prohibited.<sup>14</sup> In practice, these plans applied to one fourth of the municipalities in September 2011 (8,801 out of 36,682 according to the Ministry of Ecology) and are included in the local land-use plans. Furthermore, insurers can refuse to insure households who have built their dwelling in prohibited areas after the implementation of the plan (Insurance Code, section L 125-6), but few houses are concerned. In the United States, there is no strict prohibition but flood insurance coverage is not offered to households living in high flood risk areas (FEMA (2007)).

Mayors play a primary role in the implementation of these policies. If each jurisdiction bore the cost of its natural disasters (in particular by paying adjusted premiums to insurers), then a result à la Tiebout (1956) applies: each mayor designs the optimal red zone in his or her jurisdiction to make it most attractive. However, in practice, mayors may prioritize short-term economic and demographic development over risk prevention and may therefore minimize red zones. This phenomenon is named the *local government paradox* by Burby (2006), who provides several illustrations of it in the United States. This is one of the reasons why French and American public insurance directly link the contributions paid by households to the prevention measures taken by mayors.<sup>15</sup> Nevertheless, in France, it would appear that some local authorities exploit loopholes to delay natural risk prevention plans, as became apparent following the Xynthia Storm in 2010 (AFP (2011)).

### 3.3 Refining maps

Gross pricing opens the way to adverse selection within each tariff zone: the less exposed do not purchase insurance, so undermining mutualization. We assume here that all households purchase insurance, either because risk aversion makes the zone-based tariff remain attractive for all (even for an imprecise map) or because insurance is compulsory. In reality, in many European countries, a majority of households are not insured against floods (Bouwer et al. (2007)). In the United States, flood insurance is purchased by half of households living in flood prone areas,<sup>16</sup> and by only 1% of households living outside these areas (Dixon et al. (2006)). Important events confirm these low penetration rates for flood and earthquake insurance: “only 10% of damages were insured after the summer 2002 floods in central Europe, 3% after the 1995 Kobe earthquake, 4% after the 1999 Istanbul earthquake, 24% after the 1993 Mississippi floods, and 0.75% after the 1998 China floods” (Tallon and Vergnaud (2007), our translation).

The optimal positioning of  $n$  risk segments would be the generalization of the problem we

<sup>14</sup>See law n°95-101 of February 2, 1995 relative to the strengthening of environment protection. This law modified the law n°87-565 of July 22, 1987 relative to the organization of civil security, forest protection against fire and prevention of major risks.

<sup>15</sup>These public policies also take into account the externalities exerted by collective prevention measures (e.g. dams, levees) on neighboring jurisdictions (Grislain-Létrémy and Lemoine de Forges (2011)).

<sup>16</sup>i.e. where the flood probability is at least equal to one percent.

solved above. In the following, we privilege the analysis of the fineness of zoning and we expose a remarkably simple and powerful result.

**Definition 3.** *Zoning is the partition of space into subintervals (zones) such that building is either prohibited or authorized on each zone; the premium is uniform within each zone and actuarially balanced by zone.*

Uniform insurance with a red zone and actuarial insurance correspond to two polar examples (the latter with potentially a potentially infinite number of zones).

**Definition 4.** *Zoning  $Z_2$  is finer than zoning  $Z_1$  if every zone of  $Z_2$  is a subset of a zone of  $Z_1$  and is authorized if it belongs to an authorized zone of  $Z_1$ .*

In other words,  $Z_2$  is a further fragmentation of  $Z_1$  and building prohibition is (weakly) less restricted.

**Proposition 3.** *Refining the zoning is Pareto improving.*

*Proof.* Consider two zonings  $Z_1$  and  $Z_2$ ,  $Z_2$  being finer than  $Z_1$ .  $\hat{p}(\cdot)$  is the unique function such that, for every zone  $Z$  of the partition  $Z_2$ ,  $\hat{p}(\cdot)$  is constant over  $Z$  and equals the mean of  $p(\cdot)$  over  $Z$ . Let  $\hat{p}(\cdot)$  take the role taken by  $p(\cdot)$  in the previous analysis. Proposition 1 can be applied to  $\hat{p}(\cdot)$ , since it is decreasing: the unconstrained actuarial equilibrium for  $\hat{p}(\cdot)$  implements the optimum allocation. Clearly, this equilibrium can be implemented even if we impose zoning  $Z_2$ . Zoning  $Z_1$  being grosser than  $Z_2$ , it imposes additional constraints and thus can only lead to a Pareto inferior allocation.  $\square$

Finer zoning is more efficient in principle but is likely to be costly in terms of risk assessment. The optimal fineness is a tradeoff.

However, short-term costs and benefits are very likely to dominate the public debate and to determine the acceptance of the reform. In the short term, as people do not move or change their dwellings, they only see their insurance premium increase or decrease. In zones unchanged by the reform, inhabitants are indifferent. In refined zones, some inhabitants lose (win) because their premium increases (decreases). The State can choose the sizes of the new zones to get the reform accepted.

The State may compensate the minority losers, as the American Federal State did. Flood insurance is provided by the Federal State in the framework of the National Flood Insurance Program. The Congress established this program in 1968 and combined zone-dependent premiums with subsidies for exposed houses that were built before risk maps. This program has been more costly than expected: “at the end of 2007, [the NFIP] had borrowed \$17 billion, largely as a result of the 2004 and 2005 hurricane seasons” (Kousky and Michel-Kerjan (2010)).

“Grand Isle, Louisiana’s only inhabited barrier island, [...] has been hit by 50 major storms in the past 130 years. According to Tulane University’s Oliver Houck (Bordeau (2004)), the total federal spending in Grand Isle amounted to \$439,000 per home. Subtracting the many vacation homes increases the subsidy to \$1.28 million for each of its 622 year round residents” (Bagstad et al. (2007)).

Transition from one zoning to another certainly requires a long process of destructions and reconstructions, but in view of the economic burden of repetitive losses, [Bagstad et al. \(2007\)](#) conclude that “when evaluated from a long-term cost perspective, a one-time relocation is clearly cheaper than an ongoing cycle of damage and rebuilding”.

## 4 Industrial disasters

We will now reinterpret the model to underline the similarities and differences between natural and industrial risks. The potential losses due to households' choices (location, surface) and the probabilities have the same structure and notation. The difference is that the firm is liable for all damages. To accentuate the contrast, we assume that households are neither employees nor owners of the firm. Note that if the firm were entirely the property of the households, the issues addressed in the previous section could be readily transposed. Limited liability of the firm would certainly produce interesting effects: excessive risk-taking by the firm and, in turn, less risky location choices by imperfectly protected households.<sup>17</sup> We focus here on the “curse of unlimited liability”: fully compensated households pay no insurance premium and rent is constant across locations; households carelessly occupy the available land and, in doing so, they exert maximum land use externalities on the firm. The mayor can be seen as the head of the fund, who acts as a proxy for households and defends their interests.

First we show that location-dependent taxes set up by the mayor can play an incentive role similar to that of insurance in the previous section. Then, to complete the analysis presented for natural risks, we explore alternative scenarios about the firm's participation in the real estate market. In the simple case of red zones, we compare the maps between scenarios.

### 4.1 Location-dependent taxes

In exchange for compensation paid by the firm, the mayor could set up location-dependent taxes to harness the land use externalities. Like insurance, the tax would have a fixed part and another proportional to surface, both dependent on location. Taxes based on the hazard map reflecting the hazard function  $p(\cdot)$  implement a first-best allocation. The optimal zoning results from a tradeoff between efficiency benefits and assessment costs, the argument being similar to the one presented for natural risks.

To improve security after the AZF accident in 2001, technological risk prevention plans were created in 2003.<sup>18</sup> These plans establish red zones and zone-dependent prevention measures such as changing windows, improved air tightness in protection against toxic fumes, and thermal insulation of roofs. These renovations are mandatory up to 10% of the dwelling's market value; they typically cost €10,000 to €15,000. These measures combine technical risk reduction according to location and, in the long run, incentives for efficient land use. In their financial dimension, these measures are similar in effect to the zone-dependent taxes mentioned above,

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<sup>17</sup>Several empirical works based on the hedonic prices method show that perception of industrial risks can decrease property values ([Gawande and Jenkins-Smith \(2001\)](#), [Kiel and McClain \(1995\)](#)).

<sup>18</sup>See law n°2003-699 of July 30, 2003 relative to prevention of technological and natural risks and to damages repair. See also Environment Code, section R 562-5.

except that the community is only partially compensated by the firm for this extra burden.<sup>19</sup> This State assistance is explained by the fear that regulatory constraints and their associated cost cause firms to relocate. The European Commission decided in 2007 that this State assistance to firms did not cause a significant competition distortion.<sup>20</sup>

## 4.2 Red zones

The firm does not need land per se; it participates in the land market only to prevent the riskiest locations being occupied by potential victims. The purchasing of land by firms has been studied, but not in the framework of risk exposure. The analysis of takeover bids as in [Grossman and Hart \(1980\)](#) is certainly more advanced and it is a valid source of inspiration for solutions. [Blume et al. \(1984\)](#) and [Nosal \(2001\)](#) study the efficiency of paying compensation; [Miceli and Segerson \(2006\)](#) and [Strange \(1995\)](#) analyze the problem of holdout faced by a developer, when each individual landowner knows that each of his parcels of land is necessary for project completion and can postpone or even block the overall project.

Here the mayor represents the collective interest of households. The fact that households are landowners make them likely to benefit from exchanges, but the way competition distributes the benefits of risk reduction between stakeholders depends on market organization. In practice, land purchase can be decided principally by the firm or by the mayor.<sup>21</sup> In Moselle (France), “on the Carling petrochemical platform, one operator developed a stealth policy of buying zone 1 [the most exposed area] land and houses for sale, in order to guarantee land control in the vicinity of its installations. The dwellings are destroyed by the industrialist and the land is kept for its own use or without any determined use, guaranteeing a “buffer” function with the close neighborhoods” ([Sauvage \(1997\)](#), our translation). Mayors or the State can encourage the firm to buy or rent land exposed to industrial risks. In Waziers (France), at the end of the 1980s, “the local authority owned most of the unplowed land [...]. It obtained the industrialist’s agreement to buy [...] the land included in the future protection area within a 240 meter radius around the hydrogen storage.” ([Sauvage \(1997\)](#), our translation).

We consider three different organizations of the market offering a wide spectrum of scenarios in the distribution of bargaining power; all determine an equilibrium allocation generically denoted by  $(T^*, x^*)$  where  $T^*$  is (by convention) the net transfer from the firm to households; subscripts will refer to the particular scenario.

**Firm game.** The firm holds the bargaining power and captures all the surplus. Acceptance by the mayor depends on households doing at least as well as without the red zone. More

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<sup>19</sup>The State and mayors decided to subsidize industrialists by partly funding the purchase of the red zone and the mitigation measures required from households (order on May 3, 2007 relative to the methods of funding, follow-up and control of the implementation of land and supplementary measures foreseen by the technological risk prevention plans). These mitigation measures are currently partially subsidized by the State via a tax credit, without any transfer paid by the firm for the time being (2011 law of finance n°2010-1657 of December 29, 2010).

<sup>20</sup>See note on April 25, 2007 from the European Commission to the French authorities relative to State assistance N 508/2006.

<sup>21</sup>Industrialists cannot define a zone-dependent land use limitation. Rare exceptions can be found when industrial risks are managed by the State. For example, French law defines a zone-dependent limitation of population density by isolation polygons and areas in the very close vicinity of military pyrotechnic storage areas (law n°1929-08-08 of August 8, 1929 relative to relative to urban constraints around stores and facilities used to store, handle or produce gunpowder, ammunition, fireworks or explosives).

precisely, the firm offers a two-part tariff: it chooses the rent per unit of land and the lump-sum transfer to the community.

**Market game.** Households and the firm are both price takers. The red zone is determined by the equilibrium in the land market. The surplus is partly captured by the firm via access to the land and partly recovered by households via rents.

**Mayor game.** The mayor holds the bargaining power and redistributes to households the benefits of risk reduction extracted from the firm. The no-red-zone situation is the firm's reference for acceptance. More precisely, the mayor offers two different rents (one for households and another for the firm) and requires a lump-sum transfer from the firm to households.

The three red zones are denoted  $x_{\text{Firm}}^*$ ,  $x_{\text{Market}}^*$  and  $x_{\text{Mayor}}^*$ . We prove that all games yield a constrained optimal allocation.

The constrained optimal allocations  $(T, x)$  (Definition 2) can all be given by the maximization of the utility of households given a minimum profit of the firm  $\bar{\Pi}$ :

$$\left\{ \begin{array}{l} \max_{(z,x)} U(z, \frac{\bar{x}-x}{N}), \\ \text{s.t. } \omega + \bar{r} + \frac{T}{N} \geq z + \frac{\bar{x}-x}{N} r(x), \\ \Pi_i - T - \text{CR}(x) \geq \bar{\Pi}, \\ 0 \leq x \leq \bar{x}. \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} \max_{(T,x)} U(\omega + \frac{T}{N}, \frac{\bar{x}-x}{N}), \\ \text{s.t. } \Pi_i - T - \text{CR}(x) \geq \bar{\Pi}, \\ 0 \leq x \leq \bar{x}. \end{array} \right. \quad (9)$$

where  $\Pi_i$  is the unmodeled profit generated by the firm's primary activity. Existence and uniqueness of the solution for a given  $\bar{\Pi}$  are directly established by the argument used in the proof of Proposition 2.

We check that the equilibrium of each game corresponds to one such program.

In the mayor game, we recognize that  $\bar{\Pi} = \Pi_i - \text{CR}(0)$ . Indeed, the households capture all the benefits from risk reduction and the corresponding transfer from the firm to households is  $T_{\text{Mayor}}^* = \text{CR}(0) - \text{CR}(x_{\text{Mayor}}^*)$ .

In the firm game, the firm solves a dual program where minimum utility  $U(\omega, \frac{\bar{x}}{N})$  is guaranteed to the households.  $T_{\text{Firm}}^*$  is such that  $U(\omega + \frac{T_{\text{Firm}}^*}{N}, \frac{\bar{x}-x_{\text{Firm}}^*}{N}) = U(\omega, \frac{\bar{x}}{N})$ .

In the market game, the program cannot be directly interpreted as the equilibrium program. Existence of the equilibrium allocation is proved in Appendix A.4. Let  $r$  be the equilibrium rent in the market game. Assume for example that the market allocation is interior. We have

$$\text{MRR}(x_{\text{Market}}^*) = r, \quad (10)$$

$$\text{MRS}_{sz} \left( \omega + \frac{r x_{\text{Market}}^*}{N}, \frac{\bar{x} - x_{\text{Market}}^*}{N} \right) = r. \quad (11)$$

We take  $\bar{\Pi} = \Pi_i - r x_{\text{Market}}^* - \text{CR}(x_{\text{Market}}^*)$  in program 9. After eliminating  $T$  using the binding constraint, the first-order condition of program 9 becomes:

$$\text{MRR}(x) = \text{MRS}_{sz} \left( \omega + \frac{r x_{\text{Market}}^* + \text{CR}(x_{\text{Market}}^*) - \text{CR}(x)}{N}, \frac{\bar{x} - x}{N} \right). \quad (12)$$

By inspection of (10) and (11), we see that  $x = x_{\text{Market}}^*$  is a solution of (12). As the optimum is unique, we conclude that the market game is efficient. The line of reasoning is similar when the market yields a corner solution.



**Proposition 4.** For the three games, the equilibrium allocation  $(T^*, x^*)$  is constrained optimal and is the solution of

$$\begin{cases} x^* = 0 & \text{and } \text{MRR}(0) \leq \text{MRS}_{sz} \left( \omega + \frac{T^*}{N}, \frac{\bar{x}}{N} \right), \\ x^* \in (0, \bar{x}) & \text{and } \text{MRR}(x^*) = \text{MRS}_{sz} \left( \omega + \frac{T^*}{N}, \frac{\bar{x} - x^*}{N} \right), \\ x^* = \bar{x} & \text{and } \text{MRR}(\bar{x}) \geq \text{MRS}_{sz} \left( \omega + \frac{T^*}{N}, 0 \right), \end{cases} \quad (13)$$

where the net transfer  $T^*$  from the firm to households is

$$T_{\text{Firm}}^* \text{ such that } U \left( \omega + \frac{T_{\text{Firm}}^*}{N}, \frac{\bar{x} - x^*}{N} \right) = U \left( \omega, \frac{\bar{x}}{N} \right) \quad (14)$$

$$T_{\text{Market}}^* = rx^* \text{ where } r = \text{MRS}_{sz} \left( \omega + \frac{rx^*}{N}, \frac{\bar{x} - x^*}{N} \right), \quad (15)$$

$$T_{\text{Mayor}}^* = \text{CR}(0) - \text{CR}(x^*). \quad (16)$$

For the firm game and the mayor game, the equilibrium allocation is unique.

Comparing the  $\bar{\Pi}$  between scenarios amounts to comparing the bargaining position of the mayor in each scenario. With  $\bar{\Pi}_{\text{Nat}} = \Pi_i$ , program 9 gives the constrained-optimal solution for natural risks. This is the worst case for the community. Then come the firm game, the market game and the mayor game. Thus:  $\bar{\Pi}_{\text{Nat}} \geq \bar{\Pi}_{\text{Firm}} \geq \bar{\Pi}_{\text{Market}} \geq \bar{\Pi}_{\text{Mayor}}$ .

This suggests a strategy for comparing the red zones. As we can compare the  $\bar{\Pi}$  between scenarios, if the Engel curves of the consumers are increasing, the red zones can be readily ordered.

Simple ordinal sufficient conditions to have increasing Engel curves are:

$$\begin{aligned} \forall(z, s), \quad \frac{\partial \text{MRS}_{zs}}{\partial z} &\equiv \frac{\partial}{\partial z} \left( \frac{\partial U / \partial z}{\partial U / \partial s} \right) \leq 0, \\ \forall(z, s), \quad \frac{\partial \text{MRS}_{sz}}{\partial s} &\equiv \frac{\partial}{\partial s} \left( \frac{\partial U / \partial s}{\partial U / \partial z} \right) \leq 0. \end{aligned} \quad (\text{ENG})$$

These assumptions simply say that the relative value of the commodity becoming more abundant decreases. They are maintained hereafter. Proposition 5 compares the red zones.

**Proposition 5.** The four red zones are ordered as follows:

$$x_{\text{Nat}}^* \geq x_{\text{Firm}}^* \geq x_{\text{Market}}^* \geq x_{\text{Mayor}}^*. \quad (17)$$

*Proof.* From (9),

$$\left( \frac{d\text{MRR}}{dx} + \frac{1}{N} \frac{\partial \text{MRS}_{sz}}{\partial s} - \frac{\text{MRR}}{N} \right) \frac{dx^*}{d\bar{\Pi}} = -\frac{1}{N} \left( \frac{\partial \text{MRS}_{sz}}{\partial z} \right). \quad (18)$$

(CVX) and (ENG) are sufficient for the red zone  $x^*$  to increase with respect to  $\bar{\Pi}$ . Note that (ENG) would be sufficient with a linear constraint in program 9. As the constraint is not linear in  $x$ , we need the assumption that  $\text{CR}(\cdot)$  is convex (CVX).  $\square$

This confirms the intuition that households have more space when their situation is more favorable. The design of a red zone is an intrinsically political choice, as it depends on the distribution of the bargaining power between parties.

## 5 Delimiting red zones: the comparative statics

Red zones aptly summarize the tradeoffs encountered by decision-makers: extending the regions where building is forbidden certainly reduces the total cost of risk but squeezes households at the same time. We determine the impact of demographic evolution and climate change on optimal red zones.

For now, the increasing cost of natural disasters is largely explained by the growing urbanization of risky areas (see Introduction). However, in the future, it is presumed that climate change will increase the intensity and the frequency of natural hazards, as reiterated by the European Parliament ([Anderson \(2006\)](#)) and the Intergovernmental Panel of Climate Change ([Schneider et al. \(2007\)](#)). For example, the Netherlands are particularly vulnerable to a rise in sea level since about 70% of properties lie below either the current sea level or the river water level ([Kok et al. \(2002\)](#)). In 2008, anticipation of climate change effects led the Delta Committee to recommend several advances in water management, including land purchase along the major river areas. Industrial hazards also evolve. Materials and processes can prove more or less hazardous than previously calculated; security of installations and processes can be improved.

We will now give the comparative statics of the red zones with respect to the probability of disaster, summarized by  $\rho$ , and with respect to the exposed population, summarized by  $N$ .

In the first-best scenarios, the empty space increases with respect to  $\rho$  and can increase or decrease with respect to  $N$ . Comparative statics established by [Pines and Sadka \(1986\)](#) can be used here, since transport cost is similar to the total expected cost of risk and their theorems are sufficiently general.<sup>22</sup> The net effect of an increase in  $N$  is ambiguous, as it combines an income effect (more people receive an income) and a substitution effect (the total amount of money in circulation increases, which depreciates money relative to land).

In all the second-best scenarios, the red zone equalizes the marginal risk reduction (the MRR) to the relative value of land for the households (the  $MRS_{sz}$ ) ((8) and (13)). If the MRR becomes bigger than the  $MRS_{sz}$ , then the red zone should increase. For example, if  $\rho$  or  $N$  increases, the MRR increases, while the impact on the  $MRS_{sz}$  varies between the games. Determining the net effect requires further analysis and assumptions on risk and preferences, namely (CVX) and (ENG).

In the following analysis, we evaluate for each scenario the variations of the MRR and of the  $MRS_{sz}$  with respect to  $\rho$  (then  $N$ ). We compare the respective weights of these effects on the red zone by taking into account the preferences, the risk and the game played. Table 1 gives the signs of the derivatives of the MRR and of the  $MRS_{sz}$  with respect to  $\rho$  and  $N$  holding  $x^*$  constant (calculations are detailed in Appendix A.5). The first two lines give an immediate conclusion on the impact on  $x_{Nat}^*$  and  $x_{Firm}^*$ . These are the two cases where the aggravation of risk touches only the agents that control the red zone, namely the households with natural risks or the firm in the firm game.

To determine the variations of  $x^*$  in the other cases, consideration of the magnitude of the competing effects is necessary. Comparing the three effects in Table 1 amounts to comparing  $\partial MRR/\partial a$ ,  $\partial MRS_{sz}/\partial z \cdot \partial z/\partial a$ ,  $\partial MRS_{sz}/\partial s \cdot \partial s/\partial a$  respectively ((38) in Appendix A.5) where  $a$  stands for  $\rho$  or  $N$ . If any of these three terms is negligible with respect to at least one of the

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<sup>22</sup>[Pines and Sadka \(1986\)](#) can be applied to the case of industrial risks by choosing a selection rule (the first best is not unique): the simplest is to hold constant the transfer from the firm to the households.

Table 1: Comparative statics of  $x^*$  with respect to  $\rho$  and  $N$

	Impact on MRR	Impact on $MRS_{sz}$		Net effect on $x^*$
	$\frac{\partial MRR}{\partial a}$	through wealth	through land surface	
		$\frac{\partial MRS_{sz}}{\partial z} \frac{\partial z}{\partial a}$	$\frac{\partial MRS_{sz}}{\partial s} \frac{\partial s}{\partial a}$	$\frac{dx^*}{da}$
$a = \rho$				
Nat	+	−	0	+
Firm	+	0	0	+
Market	+	RAA	0	RAA
Mayor	+	+	0	RAA
$a = N$				
Nat	+	+	+	RAA
Firm	+	−	+	RAA
Market	+	RAA	+	RAA
Mayor	+	−	+	RAA

Note: RAA means “Requires Additional Assumptions” (see discussions in the text).

others, most cases become determined and they are easily interpreted.

If  $\partial MRS_{sz}/\partial z$  is negligible, the red zones  $x_{Market}^*$  and  $x_{Mayor}^*$  increase with respect to  $\rho$ . Note that  $\partial MRS_{sz}/\partial z = 0$  amounts to a quasilinear  $U$  (linear in  $z$ ). This means that income variations (due for example to higher rents paid by the firm) do not count for households’ renting decisions and the firm’s point of view dominates.

If  $\partial MRS_{sz}/\partial s$  is negligible, the additional land demand due to a larger population is negligible. Then the red zones  $x_{Firm}^*$  and  $x_{Mayor}^*$  increase with respect to  $N$ . In the firm game and the mayor game, the transfer per head from the firm to households decreases with respect to  $N$ , which explains their decreased demand for land. Note that  $\partial MRS_{sz}/\partial s = 0$  amounts to a quasilinear  $U$  (linear in  $s$ ).

If  $\partial MRR/\partial N$  is negligible, the red zone  $x_{Nat}^*$  decreases with respect to  $N$ . Note that  $\partial MRR/\partial N = 0$  amounts to  $\lambda_F = 0$ : only the occupied surface matters. Expected loss per head and surface per head decrease; households’ willingness to use space unambiguously increases.

In the market game, the impact of  $N$  is particularly ambiguous. Here, the net effect of an increase of rent on households’ consumption is undetermined: as renters, households want less land, but as landowners they are richer and want more. If  $\partial MRR/\partial N$  and  $\partial MRS_{sz}/\partial z$  are negligible, then the red zone  $x_{Market}^*$  decreases with respect to  $N$ . The demographic pressure on land demand dominates.

In the case of a log-log utility function and a linear loss probability, i.e.,

$$U(z, s) = \log(z) + \alpha \log(s) \quad \text{and} \quad p(x) = \rho(\bar{x} - x), \quad (19)$$

the red zones are calculable. Table 2 exposes the variations with respect to  $\rho$  and  $N$ , and the limits. As population grows, the red zone narrows down to an ultimate “risk sanctuary” (Figure 3). As the probability of disaster increases, the inhabited zone narrows down to a “city sanctuary”

or until all households are forced onto the crest. In the case of natural risks and in the firm game, the only reasonable protection for the population is that they all reach the crest. In the market and the mayor games, an interesting phenomenon appears. The community uses its bargaining power to extract rent from the firm. As the rent gets higher as risk increases, this revenue gives the households the means to rent land, even at extremely high cost. This is why the population does not squeeze itself onto the crest.

Table 2: Comparative statics with respect to risk in the case of a log-log utility function and a linear loss probability

	Variations w.r.t.		City sanctuary	Risk sanctuary
	$\rho$	$N$	$\lim_{\rho \rightarrow +\infty} x^*$	$\lim_{N \rightarrow +\infty} x^*$
$x_{\text{Nat}}^*$	$\nearrow$	$\searrow$	$\bar{x}$	$\max \left\{ \bar{x} - \frac{2\alpha}{1+\alpha} \frac{\omega}{\rho\lambda_F}; 0 \right\}$
$x_{\text{Firm}}^*$	$\nearrow$	$\searrow$	$\bar{x}$	$\max \left\{ \bar{x} - \left( \frac{2\alpha\omega\bar{x}^\alpha}{\rho\lambda_F} \right)^{\frac{1}{1+\alpha}}; 0 \right\}$
$x_{\text{Market}}^*$	$\nearrow$	$\searrow$	$\frac{1}{1+\alpha} \bar{x}$	$\max \left\{ \frac{1}{1+\alpha} \bar{x} - \frac{2\alpha}{1+\alpha} \frac{\omega}{\rho\lambda_F}; 0 \right\}$
$x_{\text{Mayor}}^*$	$\nearrow$	$\searrow$ or $\nearrow$	$\lim_{\rho \rightarrow +\infty} x_{\text{Mayor}}^* < \bar{x}$ (†)	$\max \left\{ \frac{1}{1+\alpha} \bar{x} - \frac{2\alpha}{1+\alpha} \frac{\omega}{\rho\lambda_F}; 0 \right\}$

(†) Note that  $\lim_{\rho \rightarrow +\infty} x_{\text{Mayor}}^* = \bar{x} - \frac{(1+\alpha)}{2(2+\alpha)} \frac{\lambda_F N}{\lambda_S} \left( \sqrt{1 + 4 \frac{\alpha(2+\alpha)}{(1+\alpha)^2} \frac{\lambda_S \bar{x}}{\lambda_F N} \left( \frac{\lambda_S \bar{x}}{\lambda_F N} + 1 \right)} - 1 \right)$ . If the size of the red zone increases with respect to  $N$ , the lower bound is not  $\lim_{N \rightarrow +\infty} x_{\text{Mayor}}^* = \max \left\{ \frac{1}{1+\alpha} \bar{x} - \frac{2\alpha}{1+\alpha} \frac{\omega}{\rho\lambda_F}; 0 \right\}$  but  $\lim_{N \rightarrow 0} x_{\text{Mayor}}^* = \bar{x} \left( 1 - \sqrt{\frac{\alpha}{2+\alpha}} \right)$ . Supplemental material with detailed calculations can be provided by authors upon request.

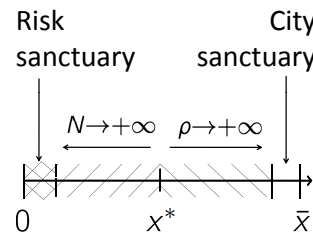


Figure 3: City and risk sanctuaries

## 6 Conclusion

The objective of our paper was to expose our views on fundamental interactions between the real estate market, insurance and the public management of natural and industrial risks. The section on natural risks was an opportunity to show how changing maps changes the economic performance of the economy. This is an essential starting point for the economic-based regulation of land use and insurance via hazard maps. In contrast, the case of industrial risks illustrated how economic forces shape risk maps. The simple case of red zones showed that climatic, technological or demographic changes are absorbed in very different ways by different

communities. Overall, it appears that the purely technical approach to designing and updating hazard maps would benefit from the addition of an economic component.

Our parallel analysis of natural and industrial disasters is all the more relevant in that the difference between natural and industrial disasters does not essentially reside in the nature of the physical phenomena. Natural disasters can cause industrial disasters. For example, in March 2011 an earthquake in Japan triggered a 33ft tsunami, which caused nuclear accidents in Fukushima. Other disasters are a combination of natural and industrial hazards, like the toxic mud floods caused by a tank failure in an aluminum plant in Hungary in October 2010. Natural and industrial hazards can mutually aggravate each other. For example, the increase of seismic activity due to the Three Gorges Dam in China, which was a subject of debate among scientists ([Naik and Oster \(2009\)](#)), is now officially recognized by Chinese authorities ([Garric \(2011\)](#)). Even past industrial accidents can worsen the consequences of natural disasters. In 2010, forest fires in Russia burned areas that were polluted after the 1986 Tchernobyl accident, recontaminating agricultural crops and local populations in 2010.

The increasingly critical question for regulators and insurers is to determine to what extent the liability of an industrialist is involved. The case of Fukushima illustrates the importance of debate on this question ([Uranaka \(2011\)](#)). Besides, scientific progress could bring new arguments to assess causality:

“companies who disproportionately contribute to global warming could be held responsible for its damage as the science of global warming becomes clearer. The most common example is owners of property damaged by a warmer world’s extreme weather suing companies that disproportionately emitted greenhouse gases” ([Moroney \(2007\)](#)).<sup>23</sup>

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<sup>23</sup>See [Ross et al. \(2009\)](#) for a legal analysis of this question.

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## A Appendices

### A.1 Notations

See Table 3.

Table 3: Principal notations

<b>Fundamental parameters and functions</b>	
$[0; \bar{x}]$	Set of inhabitable locations on the space line
$x^*$	Riskiest inhabited location (also the size of the red zone if any)
<i>Households and their preferences</i>	
$z$	Quantity of money
$s$	Housing surface
$U(z, s)$	Utility function value for $z$ and $s$ consumption
$\omega$	Income
$N$	Number of households
<i>Risk and insurance</i>	
$p(x)$	Probability of loss at $x$
$\rho$	Intensity of risk: $p(x) = \rho f(x)$
$\lambda_F$	Damage per house
$\lambda_S$	Damage per unit of surface
$CR(x^*)$	Total expected cost of risk depending on $x^*$
<b>Endogenous functions</b>	
<i>Space</i>	
$n(x)$	Density at $x$
$s(x)$	Housing surface at $x$
$r(x)$	Rent at $x$
$\bar{r}$	Redistributed rent per household
<i>Insurance</i>	
$\pi(x, s)$	Insurance premium for a housing surface $s$ at $x$
<i>Firm</i>	
$\Pi$	Firm's profit
$T$	Transfer from the firm to the households

### A.2 Necessary conditions characterizing the equilibrium

All households are identical in terms of preferences and income and they are free to move. In equilibrium,  $U^*$  is the (unique) maximum utility that can be reached given the insurance tariff.

$$\forall x \in [x^*, \bar{x}], U^* = U\left(\omega + \bar{r} - \pi_F(x) - s(x)[r(x) + \pi_S(x)], s(x)\right). \quad (20)$$

Program (HH) leads to

$$\frac{\partial U/\partial s}{\partial U/\partial z} = r(x) + \pi_S(x). \quad (21)$$

We assume here that  $p(\cdot)$  is differentiable to apply differential calculus. Let's apply the envelope theorem to the indirect utility function  $V(x)$  where  $x$  is any optimal location choice:

$$\frac{dV}{dx} = 0 = -\frac{\partial U}{\partial z} \left[ \frac{ds}{dx} \left( r(x) + \pi_S(x) \right) + s(x) \left( \frac{dr}{dx} + \frac{\partial \pi_S}{\partial x} \right) + \frac{\partial \pi_F}{\partial x} \right] + \frac{\partial U}{\partial s} \frac{ds}{dx}.$$

Thanks to (21) and since  $\partial U/\partial z \neq 0$ , we finally get

$$s(x) \left( \frac{dr}{dx} + \frac{\partial \pi_S}{\partial x} \right) + \frac{\partial \pi_F}{\partial x} = 0. \quad (22)$$

In equilibrium, a marginal reduction in rent is balanced by the marginal increase of insurance price. This expresses the tradeoff between land consumption and insurance expenditures.

Therefore an equilibrium satisfies the following necessary conditions: (20), (21), (22), (LOC), (ND or ID) and (POP).

### A.3 Proof of Proposition 1: efficiency of actuarial insurance

Let  $U_A^*$  denote the utility attained in the actuarial-insurance equilibrium. Following Fujita and Thisse (2002), we prove that the actuarial-insurance equilibrium is efficient by showing that it minimizes the social cost of achieving  $U_A^*$ .

For any allocation achieving utility  $U_A^*$ ,  $(n(x), z(x), s(x); x' \leq x \leq \bar{x})$  where  $x'$  delimits the inhabited area, the social cost for a household at  $x$  to enjoy utility  $U_A^*$  is the sum of the quantity of money  $Z(s(x), U_A^*)$  such that  $U(Z(s(x), U_A^*), s(x)) = U_A^*$  and of the cost of risk  $p(x)(\lambda_F + \lambda_S s(x))$ . Thus, we want to show that the actuarial insurance equilibrium allocation is a solution of the following program:

$$\min_{x', n(\cdot), s(\cdot)} \int_{x'}^{\bar{x}} [Z(s(x), U_A^*) + p(x)(\lambda_F + \lambda_S s(x))] n(x) dx \quad (23)$$

$$\text{s.t. } \begin{cases} \int_{x'}^{\bar{x}} n(x) dx = N, \\ \forall x \in [x'; \bar{x}], n(x)s(x) = 1. \end{cases} \quad (24)$$

Basic rearrangement gives the equivalent maximization program

$$\max_{x', s(\cdot)} \int_{x'}^{\bar{x}} \frac{\omega + \bar{r}_A - Z(s(x), U_A^*) - p(x)(\lambda_F + \lambda_S s(x))}{s(x)} dx, \quad (25)$$

$$\text{s.t. } \int_{x'}^{\bar{x}} \frac{1}{s(x)} dx = N, \quad (26)$$

where  $\bar{r}_A$  is the redistributed rent in the actuarial insurance equilibrium.

We first neglect the constraint 26. We denote

$$\psi(x, s, U_A^*) = \frac{\omega + \bar{r}_A - Z(s, U_A^*) - p(x)(\lambda_F + \lambda_S s)}{s}, \quad (27)$$

$$\Psi(x, U_A^*) = \max_s \psi(x, s, U_A^*). \quad (28)$$

The program 25 corresponds to

$$\max_{x', s(\cdot)} \int_{x'}^{\bar{x}} \psi(x, s, U_A^*) dx = \max_{x'} \int_{x'}^{\bar{x}} \Psi(x, U_A^*) dx. \quad (29)$$

As the maximum operator and  $\psi(\cdot, s, U_A^*)$  increase (as  $p(\cdot)$  decreases), by composition  $\Psi(\cdot, U_A^*)$  increases as well. We denote  $x^*$  the highest value such that  $\Psi(x^*, U_A^*) = 0$  if it exists in  $[0; \bar{x}]$  and  $x^* = 0$  otherwise. Once the objective is maximized with respect to  $s$ , one efficient value of  $x'$  is  $x^*$ .

It is straightforward that the actuarial-insurance equilibrium allocation is a solution of this rearranged program: at each  $x \geq x'$ ,  $\Psi(x, U_A^*)$  can be interpreted as the bid rent given the proposition to settle at  $x$  with a surface  $s$  and to pay the actuarial premium  $p(x) (\lambda_F + \lambda_S s)$ ;  $x^*$  can be interpreted as the most exposed inhabited area in the actuarial insurance equilibrium.

Finally, we know that the actuarial-insurance equilibrium allocation satisfies the constraint 26. Consequently, the actuarial-insurance equilibrium is efficient.

#### A.4 Part of proof of Proposition 4: existence of the market equilibrium

The proof of the existence of a market equilibrium is quite easy. The demand for land of the firm  $x(r)$  only depends on the rent. The demand of a household is  $s_d \left( w + \frac{rx(r)}{N}, r \right)$  where the first argument is the income and the second is the price.

Finding an equilibrium amounts to finding a root  $r$  to the equation

$$Ns_d \left( w + \frac{rx(r)}{N}, r \right) + x(r) = \bar{x}.$$

The LHS will be denoted  $D(r)$  henceforth.

Remark that  $D(\cdot)$  is continuous over  $\mathbb{R}$ :  $x(r)$  is continuous because  $CR(\cdot)$  is convex, which implies in turn that the households experience continuous variations of their income and of the price as  $r$  increases. Their total demand for land is therefore also continuous with respect to  $r$ .

For  $r$  very close to 0, the households have an unbounded demand for land, meaning that  $D(0^+)$  overtakes  $\bar{x}$ . For very high  $r$ , the rent paid by the firm is bounded, as it would never pay more than  $CR(0)$ . This proves that households keep a bounded income when the price of land explodes: their demand goes to 0 and  $D(r)$  is below  $\bar{x}$ .

We now use the intermediate value theorem: the previous two paragraphs establish that there is a finite  $r > 0$  such that  $D(r) = \bar{x}$ , which implies in turn that a market equilibrium exists. Note that uniqueness is not warranted.

#### A.5 Proof of comparative statics

We compute the comparative statics of  $x^*$ . In the equations below,  $a$  stands for  $\rho$  or  $N$  to economize typing.

For a given red zone  $x$ ,  $\hat{s}(x, a) = \frac{\bar{x}-x}{N}$ , and  $\hat{z}(x, a) = \omega + \frac{T(x,a)}{N}$  where

$$T(x, a) = \begin{cases} -\text{CR}(x) & \text{(Nat)} & (30) \\ T & \text{(Firm) } T \text{ such that } U\left(\omega + \frac{T}{N}, \frac{\bar{x}-x}{N}\right) = U\left(\omega, \frac{\bar{x}}{N}\right), & (31) \\ r x & \text{(Market) } r = \text{MRS}_{sz}\left(\omega + \frac{rx}{N}, \frac{\bar{x}-x}{N}\right), & (32) \\ \text{CR}(0) - \text{CR}(x) & \text{(Mayor).} & (33) \end{cases}$$

As stated by (8) and (13) for interior solutions, the red zones  $x^* \in (0, \bar{x})$  are characterized by the equality between the marginal risk reduction (MRR) and the marginal rate of substitution ( $\text{MRS}_{sz}$ ) of households.

$$\text{MRR}(x^*, a) = \text{MRS}_{sz}(\hat{z}(x^*, a), \hat{s}(x^*, a)). \quad (34)$$

By derivation of (34) with respect to  $a$ , we get

$$\left( \frac{\partial \text{MRR}}{\partial x} \frac{dx^*}{da} + \frac{\partial \text{MRR}}{\partial a} \right) = \frac{\partial \text{MRS}_{sz}}{\partial z} \frac{d\hat{z}}{da} + \frac{\partial \text{MRS}_{sz}}{\partial s} \frac{d\hat{s}}{da}. \quad (35)$$

As

$$\frac{d\hat{z}}{da} = \frac{\partial \hat{z}}{\partial a} + \frac{\partial \hat{z}}{\partial x} \frac{dx^*}{da}, \quad (36)$$

$$\frac{d\hat{s}}{da} = \frac{\partial \hat{s}}{\partial a} + \frac{\partial \hat{s}}{\partial x} \frac{dx^*}{da}, \quad (37)$$

we get

$$\frac{dx^*}{da} \left( \frac{\partial \text{MRR}}{\partial x} - \frac{\partial \text{MRS}_{sz}}{\partial z} \frac{\partial \hat{z}}{\partial x} - \frac{\partial \text{MRS}_{sz}}{\partial s} \frac{\partial \hat{s}}{\partial x} \right) = -\frac{\partial \text{MRR}}{\partial a} + \frac{\partial \text{MRS}_{sz}}{\partial z} \frac{\partial \hat{z}}{\partial a} + \frac{\partial \text{MRS}_{sz}}{\partial s} \frac{\partial \hat{s}}{\partial a}. \quad (38)$$

**With natural risks, in the firm and mayor games.** Remark that  $\partial \hat{z} / \partial x > 0$ . As  $\partial \hat{s} / \partial x = -1/N < 0$  and thanks to technical assumptions (CVX) and (ENG), the factor of  $dx^*/da$  in (38) above is negative. Therefore the sign of  $dx^*/da$  is the sign of

$$\frac{\partial \text{MRR}}{\partial a} - \underbrace{\frac{\partial \text{MRS}_{sz}}{\partial z} \frac{\partial \hat{z}}{\partial a}}_{\geq 0} - \underbrace{\frac{\partial \text{MRS}_{sz}}{\partial s} \frac{\partial \hat{s}}{\partial a}}_{\leq 0}.$$

The signs of  $\partial \text{MRR} / \partial a$ ,  $\partial \hat{z} / \partial a$  and  $\partial \hat{s} / \partial a$  are given in Table 4. Note that  $\partial \hat{z} / \partial a$  depends on the game considered. The results are presented in Table 5.

**In the market game.** (38) cannot be directly used: the sign of  $\partial \hat{z} / \partial x$  cannot be straightforwardly computed, as the rent  $r$  is endogenous.

By derivation of  $\text{MRR} = r$  and  $\text{MRS}_{sz} = r$  with respect to  $\rho$  or  $N$ , we get ambiguous expressions (available upon request).

Note: If  $\lambda_F = 0$ , then  $\partial \text{MRR} / \partial N = 0$ . Therefore, with natural risks, if  $\lambda_F = 0$ , the red zone decreases with respect to  $N$ .

Table 4: Derivatives of MRR,  $\hat{z}$  and  $\hat{s}$  with respect to  $\rho$  and  $N$

All games	$\frac{\partial \text{MRR}}{\partial \rho} = \frac{\text{MRR}}{\rho} = \frac{1}{\rho} \frac{U_s}{U_z} > 0$	$\frac{\partial \text{MRR}}{\partial N} = \frac{\lambda_F}{\bar{x}-x^*} \left( p(x^*) - \frac{\int_{x^*}^{\bar{x}} p(t) dt}{\bar{x}-x^*} \right) \geq 0$
All games	$\frac{\partial \hat{s}}{\partial \rho} = 0 = 0$	$\frac{\partial \hat{s}}{\partial N} = -\frac{\bar{x}-x^*}{N^2} < 0$
Nat	$\frac{\partial \hat{z}}{\partial \rho} = -\frac{\text{CR}(x^*)}{N\rho} < 0$	$\frac{\partial \hat{z}}{\partial N} = \frac{\lambda_S}{N^2} \rho \int_{x^*}^{\bar{x}} f(t) dt > 0$
Firm	$\frac{\partial \hat{z}}{\partial \rho} = 0 = 0$	$\frac{\partial \hat{z}}{\partial N} = -\frac{x^*}{N^2} \frac{U_s}{U_z} < 0$
Mayor	$\frac{\partial \hat{z}}{\partial \rho} = \frac{\text{CR}(0)-\text{CR}(x^*)}{N\rho} > 0$	$\frac{\partial \hat{z}}{\partial N} = -\frac{\lambda_S}{N^2} \rho \int_0^{x^*} f(t) dt < 0$

Table 5: Comparative statics of  $x^*$  with respect to  $\rho$  and  $N$

	Impact on MRR	Impact on MRS <sub>SZ</sub> through wealth	Impact on MRS <sub>SZ</sub> through land surface	Net effect on $x^*$
	$\frac{\partial \text{MRR}}{\partial a}$	$\frac{\partial \text{MRS}_{SZ}}{\partial z} \frac{\partial \hat{z}}{\partial a}$	$\frac{\partial \text{MRS}_{SZ}}{\partial s} \frac{\partial \hat{s}}{\partial a}$	$\frac{dx^*}{da}$
<i>a = ρ</i>				
Nat	+	-	0	+
Firm	+	0	0	+
Market	+	RAA	0	RAA
Mayor	+	+	0	RAA
<i>a = N</i>				
Nat	+	+	+	RAA
Firm	+	-	+	RAA
Market	+	RAA	+	RAA
Mayor	+	-	+	RAA

Note: RAA means "Requires Additional Assumptions" given in text.