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Céline Grislain-Letrémy & Sabine Lemoyne de Forges

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Céline Grislain-Letrémy[a,b,1,*], Sabine Lemoyne de Forges[c,d]

[a] *CREST-INSEE, timbre J320, 15 boulevard Gabriel Péri, 92245 Malakoff cedex, France*

[b] *University Paris-Dauphine, place du Maréchal de Lattre de Tassigny, 75775 Paris cedex 16, France*

[c] *Ecole Polytechnique, Département d'économie, 91128 Palaiseau cedex, France*

[d] *AgroParisTech ENGREF, 19 avenue du Maine, F-75732 Paris, France*

Abstract

Prevention policies against flood, such as dams or levees, are commonly designed by local jurisdictions and for most they exert externalities on neighboring jurisdictions. Each jurisdiction chooses its collective prevention effort depending on the insurance system that covers its inhabitants. As uniform insurance depends on all insureds' risk, it enables a partial integration of prevention externalities by jurisdictions. We determine under which condition uniform insurance Pareto dominates actuarial insurance.

Keywords: flood, insurance, collective prevention, interjurisdictional externalities.

1. Introduction

Risk management embraces multiple dimensions from engineering to public policy. From an economic perspective, two important aspects interact: the prevention that defines risk exposure and the financial coverage of homeowners and assets. This paper analyzes flood prevention choices by jurisdictions when these collective and observable prevention measures exert externalities on neighboring jurisdictions and when household insurance is available, especially uniform insurance. Indeed, such assumptions are adapted to flood coverage study. First, most flood collective prevention measures (dams, levees, retention basins) are observable and exert positive or negative externalities on neighboring jurisdictions (Lünenbürger, 2006). Second, in many

[☆] The conclusions and analysis set in this paper are those of the authors and do not indicate the views or opinions of their institutions.

[*] Corresponding author

Email addresses: celine.grislain.letremy@ensae.fr (Céline Grislain-Letrémy), sabine.lemoyne-de-forges@polytechnique.edu (Sabine Lemoyne de Forges)

[1] Present address: INSEE-DESE, timbre G230, 15 boulevard Gabriel Péri BP 100, 92244 Malakoff cedex, France. Tel: +33 1 41 17 60 22. Fax: +33 1 41 17 60 45

countries, flood coverage is based on uniform contributions via insurance or other redistributive systems. Thus studying flood exposure leads to a unique theoretical structure.

Our first contribution is so to build an original theoretical structure that is specifically adapted to flood exposure. Our model is based on a simple framework of two jurisdictions, one upstream and one downstream. The upstream jurisdiction, when protecting itself, may exert positive or negative externalities on the downstream one. Each jurisdiction is composed of identical agents exposed to the same completely correlated risk and can realize prevention measures to reduce the risk exposure of all its inhabitants. The household insurance scheme is initially fixed: it is either uniform State insurance or a competitive insurance market based on actuarial pricing. The two jurisdictions choose their prevention efforts given the insurance scheme and with symmetric information.

Our second contribution is to determine under which condition uniform insurance Pareto dominates actuarial insurance. Intuitively, prevention choices by jurisdictions depend on the insurance system that covers their inhabitants. As uniform insurance depends on all insureds' risk, it enables a partial integration of prevention externalities by jurisdictions. On the contrary, actuarial insurance reflects inhabitants' own risk. We show that uniform insurance Pareto dominates actuarial insurance if and only if the effect of prevention externalities is important. Note that the effect of prevention externalities for each downstream inhabitants results not only from the physical magnitude of prevention externalities but also from the size of the downstream population subject to this physical phenomenon. Model parameters are roughly calibrated to exhibit realistic cases where uniform insurance is likely to Pareto dominate actuarial insurance. Our results illustrate the general theory of second best formalized by Lipsey and Lancaster (1956-1957): "it is not true that a situation in which more, but not all, of the optimum conditions are fulfilled is necessarily, or is even likely to be, superior to a situation in which fewer are fulfilled". Here, actuarial insurance corresponds to the first best policy in the absence of externalities, but can be Pareto dominated by uniform insurance in the presence of externalities.

We revisit this way the equity-efficiency tradeoff between actuarial and uniform insurance in the presence of prevention externalities. Picard (2008) compares actuarial and uniform insurance against natural disasters in the absence of externalities. The author shows that, under imperfect information about prevention costs, actuarial insurance provides incentives for prevention and is so efficient, but generates strong inequalities between individuals with different risk types and prevention costs; on the contrary, uniform insurance is inefficient but ensures equity. Our finding establishes that, in the presence of another market imperfection (prevention externalities), the tradeoff is not always at stake: uniform insurance is more efficient than actuarial insurance if and only if the effect of prevention externalities is important.

We now detail and illustrate the specificities of flood risk and why the set of assumptions used by other papers studying prevention are not suited to flood risk analysis.

The flood issue. Our paper contributes to analyzing a practical problem that is flood exposure. Floodplains, deltas, and coastal areas development has historically been driven by the hydraulic works as observed on the Mississippi, Rhine, Nile or Mekong rivers. Dams, levees, elevations or polders were either designed to expand the territories available for development, to get water storage for dry seasons, or to simply protect from floods or submersion (Fanchette, 2006).

These collective prevention actions are observable. Not only they can be easily identified by neighbors, but information about hydraulic works, like damns and levees, can be spread at a much larger scale. Since 1928 the International Commission on Large Dams assesses and inventories large dams. Levees inventory is more recent. In the United States, Congress created the National Committee on Levee Safety in 2007. In France, an important program of inventory of levees protecting against flood has been set up in 1999 (Mériaux et al., 2003).

On rivers and deltas, flood protection works that are built can have important impacts on the flood exposure of other areas alongside the water. Indeed, dams and levees built by a jurisdiction create positive or negative externalities on its neighbors, upstream or downstream, during the normal course of their operations (Tobin, 2007). A dam built by an upstream jurisdiction protects the downstream ones from flooding. On the contrary, an upstream levee increases the downstream flow. In the event of dam or levee failures, negative externalities are exerted downstream as a failure causes a large increase in velocity and flow rate. The Three-Gorges Dam in China is a typical example: it changed the whole hydraulic of the Yangtse river in its regular regime downstream as well as upstream, and in the case of flood it provides a regulation of flows very different from what was in place beforehand (Zhu and Rong, 2010). More generally, land use choices are an important part of floodplain management and may change the exposure of areas along the river. For instance, waterproofing with no proper dimensioning of water evacuation can create negative externalities, as recalled by the 2010 flash floods and mudslides in Madeira. The existence of such externalities pleads for the coordination of prevention policies.

Risk managers, and particularly engineers, have known for a long time the importance of integrating collective prevention externalities when designing flood control systems. Some examples reveal strong efforts by central government to coordinate local prevention policies and reduce negative prevention externalities this way. In the Netherlands, construction and maintenance of "polders" have been performed by the "waterschappen" (local water communities) since the 18th century. Because of the numerous legal disagreements between polders due to their induced negative externalities - typically soil removed to strengthen levees -, the "waterstaat" (central water administration) was created in 1798 to coordinate all these local activities. Other examples illustrate efforts by central government to compensate neighboring inhabitants for suffering negative prevention externalities. In France, some projects include initial compensation to neighboring inhabitants for disturbance, loss of real estate value and also the determination of ex post compensation for flooding.[2]

However, in most cases, central government faces important difficulties in coordinating these local prevention measures. In the case of the 1993 Missouri floods for instance, some landowners had built higher levees than authorized to protect their crops. Therefore they put other developed land nearby at risk (Rasmussen, 1999). Indeed, in practice central government can rarely en-

[2]An example is the dynamic reduction in flooding of the Meuse at Mouzon in France (Chambre d'Agriculture des Ardennes, 2006).

force prevention constraints on jurisdictions. Even if legal sanctions exist, these may not have the desired preventive effect on jurisdictions. In the United States, states can be sued if protection measures are not implemented: for example "the Legislature approved $500 million in settlements of claims in 2005 for failed levees in the 1986 and 1997 floods" against the state of California (California Hearing, 2005).

Central government is concerned by collective prevention not only because of civil security but also because it takes in charge the compensation in the case of natural disasters for individuals and jurisdictions (via insurance coverage, financial aids, conditional loans and/or urgency disaster rescue). In many countries, flood coverage is based on uniform contribution. Some countries such as France or Spain have set up bundled uniform insurance for natural disasters.[3] Many other countries have designed redistributive systems that are similar to uniform insurance. In Italy or Korea, compensation for floods is based on public assistance; in Australia, Canada, Germany or in the Netherlands, public funds coexist with private flood insurance and provide important compensation because insurance penetration rate is low (Dumas et al., 2005).[4]

Insurance pricing can take into account collective prevention or mitigation measures, as these measures are observable. In France, for example, if a jurisdiction has been touched by several floods and if no risk prevention plan has been undertaken by the jurisdiction, inhabitants' insurance deductibles are significantly increased (Dumas et al., 2005). It is also the case in the United States, where flood insurance is actuarial (with subvention of specific risks): in the framework of the National Flood Insurance Program, insured households receive a rebate on their premium depending on prevention measures taken by their jurisdiction (Burby, 2001).

For all these reasons, we focus on observable collective prevention with risk externalities and uniform insurance to study flood exposure.

Related literature. To model the link between flood prevention and coverage policies at the jurisdictional level, we were naturally driven to consider specific assumptions: asymmetric prevention externalities and risk correlation within one jurisdiction. Scrutinizing the uniform insurance case was especially interesting given the policies in place in many countries. As underlined by Lipsey and Lancaster (1956-1957), an adequate and specific model is required to assess and compare second best policies. To our knowledge, the specifications of other papers that study prevention are adapted to generic issues or to other risks. Most of them consider actuarial insurance and individual prevention with asymmetric information and, for some of them, without risk externalities. This is why they cannot be transposed to the analysis of our subject: flood collective prevention and uniform coverage.

[3] In France and Spain, insurance against natural disasters is a mandatory guarantee of classical home insurance. In France, the State provides its unlimited guarantee to the natural disasters insurance system via the Caisse Centrale de Réassurance and regulates the price of natural disasters insurance. In Spain, insurance against natural disasters is provided by the Consorcio de Compensación de Seguros.

[4] In Germany, for example, less than 10% of households have purchased flood insurance (Bouwer et al., 2007) and public aid to households and businesses reached Bn€ 1.7 following the Elbe floods in 2002 (Dumas et al., 2005).

Some generic models study incentive tools for individual prevention with risk externalities and consider actuarial insurance. Hofmann (2007) analyzes agents with interdependent risks and investigates the case of a benevolent monopolist insurer. She shows that under actuarial insurance with imperfect information, the insurer can reach the social optimum by engaging in price discrimination as it reduces the cost of risk selection. Muermann and Kunreuther (2008) consider actuarial insurance and positive externalities; they show the under-investment in self-protection in the absence of coordination among the individuals.

The study of specific risks has lead to different assumptions that also consider actuarial insurance. Lakdawalla and Zanjani (2005) address the specific case of terrorism, where self-protection induces negative externalities as risk terrorists will switch to more vulnerable targets. The authors consider actuarial insurance with a loading factor and asymmetric information. They show that government subsidies for terror insurance limit self-protection. Lohse et al. (2012) analyze local public goods providing self-protection or self-insurance and targeting uncorrelated risks. They consider actuarial insurance and show that the availability of market insurance reduces the provision level of the public good for both public and private provision. In the fiscal federalism literature, Persson and Tabellini (1996) analyze the tradeoff between federal risk sharing and moral hazard under asymmetric information; they do not consider risk correlation between local entities.

Most natural disasters, such as storms or earthquakes, do not imply prevention externalities. In the absence of externalities, Picard (2008) compares actuarial and uniform insurance against natural disasters. The author illustrates the equity-efficiency tradeoff for the coverage of natural disasters: incentives to individual prevention through insurance create strong inequalities between individuals with different risk types and prevention costs, but actuarial insurance combined with tax-transfers overcomes this tradeoff.

Flood coverage has been specifically studied by Lünenbürger (2006). The author considers flood collective prevention as public goods with unidirectional spillovers and under actuarial insurance. He focuses on the supply of flood prevention as the outcome of voting procedure and he compares different federal settings. But he does not address collective prevention choices with respect to insurance.

Our paper is organized as follows. Section 2 lays out the model assumptions. Section 3 presents the first best, that is the situation where prevention choices are centralized. Section 4 presents the situations where prevention choices are chosen by jurisdictions in complete autarky, either under uniform or actuarial insurance. Section 5 compares these second bests (autarky) with the first best (centralization). Section 6 derives the comparison between actuarial and uniform insurance systems in autarky and then determines under which uniform insurance Pareto dominates actuarial insurance. Section 7 concludes.

2. Model

Jurisdictions. We consider a watershed composed of two jurisdictions 1 and 2 located next to a river and a central government. Jurisdiction 1 is upstream; jurisdiction 2 is downstream. The jurisdictions respectively consist of a population of N_1 individuals and of N_2 individuals. All individuals are identical. We denote

$$N_2 = \eta N_1. \tag{1}$$

We denote W_j the total wealth in jurisdiction j, $j \in \{1, 2\}$.

Each jurisdiction can organize wealth redistribution between its inhabitants. Similarly, central government can organize wealth redistribution between jurisdictions and implement it via transfers. Therefore, we do analyze here inequalities or redistribution effects neither inside jurisdictions nor between them.

Individual preferences. Preferences of the inhabitants in jurisdiction j are described by a common utility function $u(x_j)$ where x_j is the private good consumption. We assume that individuals are risk averse and therefore that $u(\cdot)$ is increasing and concave.

Flood risk. Each individual has an income I and is exposed to a loss L.[5] We assume that flood risks are perfectly correlated within a jurisdiction: one flood may damage all inhabitants in a jurisdiction or nobody. However, when a flood occurs next to the river, each jurisdiction may be flooded or not depending on its prevention effort. We assume that the initial loss probability is the same for both jurisdictions and p^0 denotes this common probability.

Flood collective prevention. Each jurisdiction can reduce its expected loss by implementing preventive or protective measures, henceforth called prevention, to reduce the risk exposure for all its inhabitants. Since it modifies risk exposure of neighboring inhabitants in a nonrival and nonexcludable way, flood collective prevention is a local public good.

The prevention effort of jurisdiction j is denoted e_j. It has a cost denoted by $C_j(e_j)$ which is assumed to be quadratic:

$$C_1(e_1) = \frac{c_1}{2} e_1^2, \quad c_1 > 0, \tag{2}$$

$$C_2(e_2) = \frac{c_2}{2} e_2^2, \quad c_2 > 0. \tag{3}$$

Prevention is funded at the jurisdictional level by lump sum local taxes: collective prevention benefits to all inhabitants of the jurisdiction and is equally funded by each of them.[6]

We abstract from consideration of voting procedure to choose prevention within a jurisdiction.[7] We simply consider that the local prevention choice maximizes the total wealth of the inhabitants.

[5]Considering different losses among individuals within a jurisdiction and allowing transfers between them would not modify our results.

[6]Note that taxes providing prevention incentives to individuals would not make sense, as individuals cannot decide the collective prevention effort on their own.

[7]See Lünenbürger (2006) for a study of supply of flood prevention as the outcome of a voting procedure.

Flood prevention externalities. Jurisdiction 1's final expected loss depends on its own prevention effort. Jurisdiction 2 can also decrease its expected loss by its own prevention effort. However, it is subject to loss externalities originating from the prevention effort implemented upstream by jurisdiction 1. Therefore the expected losses of all inhabitants are correlated. We use ϵ to denote the externalities coefficient. For the sake of simplicity, we assume that the expected losses at the jurisdictional scale are[8]

$$EL_1(e_1) = N_1 p^0 L(1 - e_1), \tag{4}$$

$$EL_2(e_1, e_2) = N_2 p^0 L(1 - e_2 - \epsilon e_1). \tag{5}$$

If $\epsilon > 0$, the prevention effort by jurisdiction 1 reduce jurisdiction 2's expected loss. Thus, $\epsilon > 0$ corresponds to positive externalities and $\epsilon < 0$ to negative ones. Note that the form chosen for externalities implies that jurisdiction 2 cannot reduce the impact of the externalities generated by jurisdiction 1 using its own prevention effort. Besides, this specification allows to consider prevention effort as decreasing the loss probability (self-protection in the sense of Ehrlich and Becker (1972)) or as decreasing the potential loss (self-insurance in the sense of Ehrlich and Becker (1972)). Under full insurance, these two interpretations are equivalent.

To guarantee expected losses to be positive, we assume that[9]

$$e_1 \leq 1, \tag{6}$$

$$e_2 + \epsilon e_1 \leq 1. \tag{7}$$

Note that we allow negative prevention efforts: a jurisdiction can build a construction work alongside the river that increases its expected loss.[10]

Household insurance. This model represents a region with two connected jurisdictions among which risks are correlated. However, at the national level, the number of regions increases risk tolerance and diversifies risk, so that we can assume risk neutrality of the insurance sector. Household insurance can be provided by a competitive market or an efficient administration. For the sake of simplicity, there are no administrative costs (no loading factor). We assume that all individuals purchase insurance for reasons exposed thereafter.

We assume full insurance and consider two different insurance schemes: uniform or actuarial insurance.

We consider compulsory uniform insurance as implemented in several countries. Uniform insurance has to be understood here in a broad meaning; it includes other solidarity mechanisms than insurance itself, and especially public aid: State assistance is funded by taxes which are

[8]This expression is different from the one used by Hofmann (2007) and Muermann and Kunreuther (2008). Both papers assume that a loss directly caused by an agent and a loss indirectly caused via others are independent.

[9]These conditions of validity are derived in AppendixA in each studied case.

[10]Under self-protection, the prevention efforts have also to guarantee a loss probability lower than 1, that is $p^0(1 - e_1) \leq 1$, $p^0(1 - e_2 - \epsilon e_1) \leq 1$.

similar to insurance premiums. We consider that all individuals benefit from a compensation after natural disasters at a uniform price organized by the State.[11]

The uniform premium Π^u depends on the prevention efforts of both jurisdictions.

$$\Pi^u(e_1, e_2) = \frac{N_1 p^0 L(1 - e_1) + N_2 p^0 L(1 - e_2 - \epsilon e_1)}{N_1 + N_2},$$
$$= p^0 L\left(1 - \frac{1 + \eta\epsilon}{1 + \eta} e_1 - \frac{\eta}{1 + \eta} e_2\right). \tag{8}$$

The effect of prevention externalities appears here as $\eta\epsilon$. Consistently, it depends on the externality coefficient ϵ and on the relative size η of the population that is subject to these externalities. The uniform premium shares the total cost of risk and externalities between the two jurisdictions. The two factors $1/(1 + \eta)$ and $\eta/(1 + \eta)$ that appear in the expression traduce this *loss sharing effect*, respectively for jurisdiction 1 and for jurisdiction 2.

We also consider actuarial insurance. Competition implies that insurance makes individuals pay for their own risk. It does not make individuals living in jurisdiction 1 pay for the externalities their collective prevention exert on jurisdiction 2. Indeed, if externalities are negative, individuals will not purchase a more expensive insurance that internalizes the externalities they create. If externalities are positive, an insurer is able to offer a reduced premium only if he insures all the individuals living in jurisdiction 2; this condition is not compatible with competition.

Therefore, under actuarial insurance, the premiums are

$$\Pi_1^a = p^0 L(1 - e_1), \tag{9}$$
$$\Pi_2^a = p^0 L(1 - e_2 - \epsilon e_1), \tag{10}$$

and all individuals purchase flood insurance.

Timing. Central government and jurisdictions have symmetric information. The timing of the model is as follows.

Stage 1: The insurance scheme is fixed. Central government chooses the form of the prevention incentives, if any, and the transfers policy between jurisdictions.

Stage 2: Jurisdictions determine their prevention efforts.

Stage 3: The state of nature is realized: losses are revealed and each individual knows its final wealth.

[11] Certainly, Coate (1995) explains that the equivalence between *ex post* taxation and uniform insurance is imperfect. Ex post assistance by the State is less efficient because assistance may rely on approximate loss assessments or discretionary decisions. Besides, as natural disasters assistance is provided by various actors (non-profit organizations, States), the uninsured can free-ride. We leave these issues aside.

As in Hofmann (2007) and Muermann and Kunreuther (2008), we consider the Nash equilibrium in Stage 2. However, due to our specification (Equations 4 and 5), the prevention effort of one jurisdiction does not depend on the prevention effort of its neighbor. Indeed, jurisdiction 1 is not impacted by jurisdiction 2's prevention effort, and jurisdiction 2 cannot use its own prevention effort to reduce the impact of the externalities exerted by jurisdiction 1.

3. First best: centralization

We analyze here the first best situation, where central government decides the prevention efforts and simultaneously designs a transfers policy between the two jurisdictions. Under full uniform ($s = u$) or actuarial ($s = a$) insurance and in the presence of transfers, the program of the central government is to maximize the sum of the individuals' wealth:

$$\max_{e_j^s} W_1 + W_2 = N_1 \left(I - \Pi_1^s - \tfrac{c_1}{2N_1} (e_1^s)^2 \right) + N_2 \left(I - \Pi_2^s - \tfrac{c_2}{2N_2} (e_2^s)^2 \right). \tag{11}$$

This leads to the following prevention efforts:

$$e_1^{**} = \frac{p^0 L N_1}{c_1}(1 + \eta\epsilon), \tag{12}$$

$$e_2^{**} = \frac{p^0 L N_2}{c_2}. \tag{13}$$

The prevention effort in jurisdiction 1 internalizes externalities exerted on jurisdiction 2.

Note that the prevention efforts do not depend on the given insurance system whether uniform or actuarial. Indeed, the sum of expected losses supported by the jurisdictions does not depend on the way the financial burden for flood losses is shared between jurisdictions. However, even if the jurisdictions have the same population size and the same cost function, the two insurance systems do not lead to the same wealth in each jurisdiction because of the geographic heterogeneity between both jurisdictions. These wealths are detailed in AppendixB.

If there was a benevolent monopolist insurer and if insurance was mandatory, these prevention efforts could be implemented by the following insurance mechanism: the unique insurer would make inhabitants pay for the overall consequences of their prevention efforts, that is for their own risk and for the prevention externalities they exert on the neighboring jurisdiction.

$$\Pi_1^i = p^0 L (1 - (1 + \eta\epsilon) e_1), \tag{14}$$

$$\Pi_2^i = p^0 L (1 - e_2). \tag{15}$$

Note that assuming mandatory insurance is required in the case of negative externalities with a downstream jurisdiction more populated than the upstream one. In that case, the term $\eta\epsilon e_1$ in Equation 14 strongly increases the premium offered to upstream households; despite of their risk aversion, these households could so prefer not to purchase insurance.

4. Second best: autarky

We now assume that central government does not coordinate local prevention policies in any way. In this second best world that we call autarky, we consider uniform insurance that depends on all insureds' risk and partially integrates prevention externalities; we also consider actuarial insurance that makes individuals pay for their own risk but not for the prevention externalities they exert on the neighboring jurisdiction.

Each jurisdiction maximizes the wealth of its inhabitants either under uniform insurance ($s = u$) or under actuarial insurance ($s = a$).

$$\forall j \in \{1; 2\}, \max_{e_j^s} W_j^s = N_j \left(I - \Pi_j^s - \frac{c_j}{2N_j}(e_j^s)^2 \right). \tag{16}$$

Uniform insurance. The uniform premium provides a price signal on the direct impact of prevention as well as on the externalities created for the upstream jurisdiction. However, the signal on these impacts is diluted since the uniform premium shares the total cost of risk and externalities between the two jurisdictions. The factors $1/(1 + \eta)$ and $\eta/(1 + \eta)$ that respectively appear in the expression for the prevention effort by jurisdiction 1 and by jurisdiction 2 reflect this loss sharing effect:

$$e_1^{u*} = \frac{p^0 L N_1}{c_1} \frac{1 + \eta\epsilon}{1 + \eta} < e_1^{**}, \tag{17}$$

$$e_2^{u*} = \frac{p^0 L N_2}{c_2} \frac{\eta}{1 + \eta} < e_2^{**}. \tag{18}$$

Because of the loss sharing effect, prevention efforts are lower than those that would be implemented by central government under centralization.

Actuarial insurance. Under actuarial insurance, the prevention effort in jurisdiction 1 does not internalize externalities exerted on the downstream jurisdiction 2, since actuarial insurance does not give any price signal on these.

$$e_1^{a*} = \frac{p^0 L N_1}{c_1} < e_1^{**} \Leftrightarrow \epsilon > 0, \tag{19}$$

$$e_2^{a*} = \frac{p^0 L N_2}{c_2} = e_2^{**}. \tag{20}$$

This is why centralization leads to larger prevention efforts than in autarky if and only if externalities are positive. Note that, in autarky, in the absence of externalities, actuarial insurance leads to optimum prevention efforts.

In each jurisdiction, the wealths under uniform and actuarial insurance are detailed in AppendixC.

5. Comparing autarky and centralization

In autarky, the prevention efforts are not optimum. However, modifying them would reduce the welfare of jurisdictions. To avoid this, central government can organize transfers between jurisdictions. As centralization corresponds to the first best and includes a transfers policy, it Pareto dominates the second best, either under uniform or actuarial insurance. It is so straightforward that the social welfare in centralization is larger than the social welfare in autarky.

Uniform insurance. Using AppendixB and AppendixC, the gain in social welfare due to centralization is

$$W_1^{u**} + W_2^{u**} - W_1^{u*} - W_2^{u*} = \frac{(p^0 L N_1)^2}{2} \left(\frac{\eta}{1+\eta} \right)^2 \left(\frac{(1+\eta\epsilon)^2}{c_1} + \frac{1}{c_2} \right) > 0. \tag{21}$$

The social welfare difference increases with the loss sharing effect (expressed by $\eta/(1+\eta)$), which is the difference between the prevention efforts in autarky and the optimum efforts. Indeed, centralization corrects the loss sharing effect, and each jurisdiction benefits from this adjustment. Note that transfers between the two jurisdictions are unnecessary to increase the wealth of each of them, unless the two cost coefficients are very disproportionate (AppendixD).[12]

Actuarial insurance. Similarly, the gain in social welfare due to centralization derives from AppendixB and AppendixC:

$$W_1^{a**} + W_2^{a**} - W_1^{a*} - W_2^{a*} = \frac{(p^0 L N_1)^2}{2c_1} \eta^2 \epsilon^2 \geq 0. \tag{22}$$

In the absence of prevention externalities ($\epsilon = 0$), actuarial insurance in autarky leads to the optimum prevention efforts. Note that transfers between the two jurisdictions are here necessary to increase the wealth of each of them: as the first best implies the internalization of externalities, it penalizes jurisdiction 1 and benefits to jurisdiction 2 (AppendixD).[13]

[12] For example, in the absence of prevention externalities, transfers between the two jurisdictions are necessary to increase the wealth of each of them if and only if $c_1/c_2 < 1/2$ or $c_1/c_2 > 2$ (AppendixD).

[13] Note that central government could implement first-best prevention efforts and welfare by setting up Pigouvian taxes for jurisdictions. As we consider full insurance, this policy is equivalent to modifying the households insurance premium in order to implement the insurance pricing described in Section 3.

6. Comparing uniform and actuarial insurance

6.1. Comparing uniform and actuarial insurance in autarky

In autarky, the prevention efforts are lower under uniform insurance than under actuarial insurance, except for jurisdiction 1 in the case of large positive externalities:

$$e_1^{u*} < e_1^{a*} \Leftrightarrow \epsilon < 1, \tag{23}$$
$$e_2^{u*} < e_2^{a*}. \tag{24}$$

Indeed, prevention is generally of more benefit under actuarial insurance because of the loss sharing effect. But, for jurisdiction 1, this effect is at stake if externalities are not too positive only. Otherwise, partial internalization of externalities under uniform insurance leads jurisdiction 1 to a larger prevention effort, despite the loss sharing effect.[14]

These differences of prevention efforts explain why uniform insurance leads each jurisdiction j to a different wealth (W_j^{u*}) from the one under actuarial insurance (W_j^{a*}):

$$W_1^{u*} - W_1^{a*} = N_1(\Pi_1^a(e_1^{a*}) - \Pi^u(e_1^{u*}, e_2^{u*})) + C_1(e_1^{a*}) - C_1(e_1^{u*}), \tag{25}$$
$$W_2^{u*} - W_2^{a*} = N_2(\Pi_2^a(e_1^{a*}, e_2^{a*}) - \Pi^u(e_1^{u*}, e_2^{u*})) + C_2(e_2^{a*}) - C_2(e_2^{u*}). \tag{26}$$

It clearly appears that the difference of wealth in each jurisdiction depends on the difference in its own prevention effort and also depends on the other jurisdiction's prevention effort. Indeed, as uniform insurance premium depends on the prevention efforts of both jurisdictions, it enables inhabitants of one jurisdiction to benefit from the prevention realized by the other jurisdiction. For jurisdiction 2, its wealth under actuarial insurance also depends on jurisdiction 1's prevention effort because of externalities.

We can now derive these equations under our specification and determine the conditions under which uniform insurance leads jurisdictions to a larger wealth.

Jurisdiction 1. Uniform insurance leads jurisdiction 1 to a larger wealth than actuarial insurance if and only if

$$W_1^{u*} - W_1^{a*} = \frac{(p^0 L N_1)^2}{c_1} \frac{\eta}{(1+\eta)^2} \left(\eta^2 \frac{c_1}{c_2} + \eta \frac{\epsilon^2 - 1}{2} + \epsilon - 1 \right) \geq 0. \tag{27}$$

Therefore,

$$\forall \epsilon, \exists \hat{\eta}_1 / \forall \eta > \hat{\eta}_1, W_1^{u*} > W_1^{a*}. \tag{28}$$

[14] Specifically, under actuarial insurance, a marginal increase in each jurisdiction's prevention effort decreases the premium of its inhabitants by 1. Under uniform insurance, for jurisdiction 1, it decreases the premium by $(1 + \eta\epsilon)/(1 + \eta)$, which is strictly less than 1 if and only if $\epsilon < 1$; for jurisdiction 2, it decreases the premium by $\eta/(1 + \eta) < 1$.

Figure 1 illustrates this result. It compares jurisdiction 1's wealth under uniform and actuarial insurance in the case where $p^0 LN_1/c_1 = 0.1$ and with equal cost coefficients.[15] Curve L_1 on Figure 1 corresponds to the set of points (ϵ, η) such that the wealth of jurisdiction 1 is the same under uniform and actuarial insurance ($W_1^{u*} = W_1^{a*}$). Either right to this curve or above it, that is either for small η and large ϵ or for large η, uniform insurance leads jurisdiction 1 to a larger wealth than actuarial insurance ($W_1^{u*} > W_1^{a*}$).

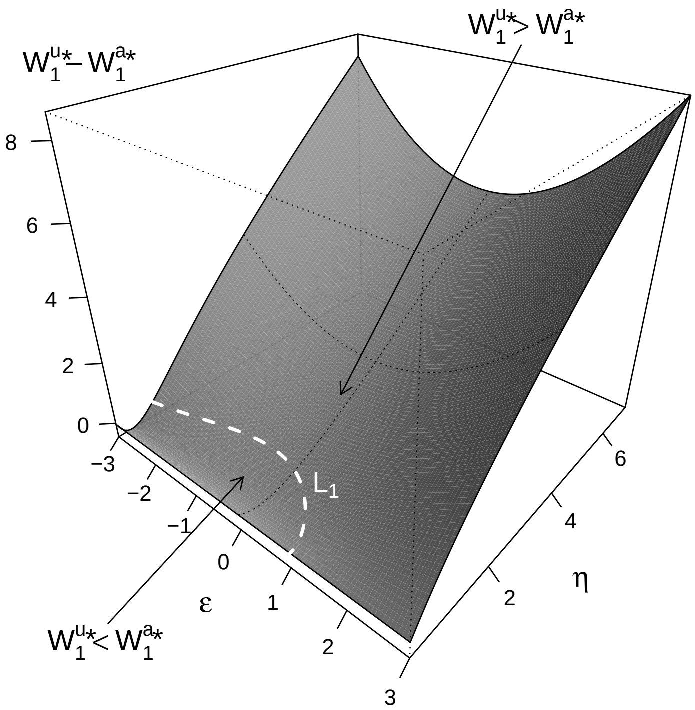

Figure 1: Comparative statics of the difference of jurisdiction 1's wealth between uniform and actuarial insurance ($p^0 LN_1/c_1 = 0.1$, $c_1 = c_2 = 10$)

Intuitively, for small η that is when jurisdiction 2 is sparsely populated by comparison with jurisdiction 1, what mainly matters for jurisdiction 1, even under uniform insurance, is its own prevention effort. Thus, jurisdiction 1 is better off under uniform insurance if and only if uniform insurance rewards more its prevention effort than actuarial insurance does, that is if and only if externalities are strongly positive.[16]

On the contrary, for large η that is when jurisdiction 2 is relatively densely populated, jurisdiction 1 strongly benefits from jurisdiction 2's prevention effort under uniform insurance. Thus,

[15] Note that the whole set of points (ϵ, η) considered on Figure 1 verify the conditions of validity under uniform and actuarial insurance (AppendixA).

[16] Specifically, for small η, jurisdiction 1's wealth is larger under uniform insurance than under actuarial insurance if and only if $\epsilon \geq 1$. Consistently, in that case, jurisdiction 1's prevention effort is larger under uniform insurance (Equation 23).

jurisdiction 1 is better off under uniform insurance, whether it makes more or less prevention than under actuarial insurance. Note that the impact of jurisdiction 2's prevention effort on jurisdiction 1's wealth is important inasmuch as this effort is important, and so inasmuch as the prevention cost for jurisdiction 2 is small (Equation 27).

Jurisdiction 2. The situation for jurisdiction 2 is not as simple because of borne externalities. Uniform insurance leads jurisdiction 2 to a larger wealth than actuarial insurance if and only if

$$W_2^{u*} - W_2^{a*} = \frac{(p^0 L N_1)^2}{c_1} \frac{\eta}{(1+\eta)^2} \left(\eta^2 \left(\epsilon(\epsilon - 1) - \frac{c_1}{c_2} \right) - \eta \frac{c_1}{2c_2} + 1 - \epsilon \right) \geq 0. \tag{29}$$

Therefore,

$$\exists \hat{\eta}_2 / \forall \eta > \hat{\eta}_2, W_2^{u*} > W_2^{a*} \Leftrightarrow \epsilon(\epsilon - 1) - \frac{c_1}{c_2} > 0. \tag{30}$$

The comparative statics for jurisdiction 2 are illustrated on Figure 2 for the set of parameters previously used ($p^0 L N_1 / c_1 = 0.1$ and equal cost coefficients).[17] Curves L_2 correspond to the set of points (ϵ, η) such that the wealth of jurisdiction 2 is the same under uniform and actuarial insurance ($W_2^{u*} = W_2^{a*}$). Outside the area delimited by these two curves, that is either for small η and small ϵ or for large η and large $|\epsilon|$, uniform insurance leads jurisdiction 2 to a larger wealth than actuarial insurance ($W_2^{u*} > W_2^{a*}$).

Intuitively, when jurisdiction 2 is sparsely populated by comparison with jurisdiction 1, that is for small η, what mainly matters even for jurisdiction 2 is jurisdiction 1's prevention effort. Under actuarial insurance, this effort impacts jurisdiction 2 only through the direct effect of externalities, and jurisdiction 2 benefits from this effort if and only if externalities are positive. Under uniform insurance, the small jurisdiction 2 strongly benefits from jurisdiction 1's effort thanks to the loss sharing effect. Thus, for small η, jurisdiction 2's wealth is larger under uniform insurance than under actuarial insurance if and only if externalities are not too strongly positive.[18]

On the contrary, when jurisdiction 2 is relatively densely populated, that is for large η, under uniform insurance, the loss sharing is certainly not in favor of jurisdiction 2, but uniform insurance leads to a partial internalization of externalities by jurisdiction 1, which is desirable inasmuch as externalities are negative or strongly positive. Note that this impact is important inasmuch as c_2 is lower than c_1 (Equation 29).

These results lead to the following proposition.

[17] Note that the whole set of points (ϵ, η) considered on Figure 2 verify the conditions of validity under uniform and actuarial insurance (AppendixA).

[18] Specifically, for small η, jurisdiction 2's wealth is larger under uniform insurance than under actuarial insurance if and only if $\epsilon < 1$ (Equation 29).

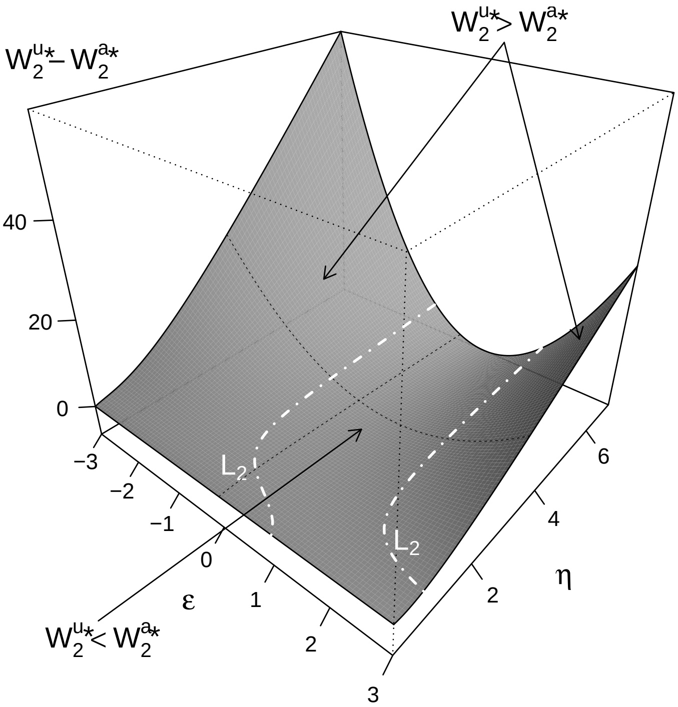

Figure 2: Comparative statics of the difference of jurisdiction 2's wealth between uniform and actuarial insurance ($p^0 LN_1/c_1 = 0.1$, $c_1 = c_2 = 10$)

Proposition 1. *Each jurisdiction is better off under uniform insurance for and only for large η and large ϵ in absolute value: uniform insurance makes jurisdiction 1 benefit from loss sharing and jurisdiction 2 from partial internalization of prevention externalities.*

Proof. This derives from Equations 28 and 30. □

We have so determined the conditions under which each jurisdiction is better off under uniform insurance, while considering the two jurisdictions in autarky. We now consider that central government can organize transfers between jurisdictions and determine the condition under which uniform insurance Pareto dominates actuarial insurance. As this condition is by definition less restrictive than the previous ones, we will specify the cases where Pareto dominance of uniform insurance requires transfers between the two jurisdictions.

6.2. *Pareto dominance of uniform insurance*

As there are only two jurisdictions, the existence of a transfers policy such that there is Pareto dominance is equivalent to an increase of social welfare. Indeed, the decrease of wealth in one jurisdiction can be compensated by a transfer from the other jurisdiction if and only if the wealth loss in one jurisdiction is lower than the wealth gain in the other, that is if and only if social welfare increases.

This leads to the following proposition, where SW^u denotes the social welfare under uniform insurance and SW^a the one under actuarial insurance.

Proposition 2. *As uniform insurance partially internalizes prevention externalities, it Pareto dominates actuarial insurance inasmuch as the effect of externalities is important:*

- *uniform insurance Pareto dominates actuarial insurance inasmuch as the physical magnitude ϵ of prevention externalities is important in absolute value.*

$$\forall \eta, \exists \bar{\epsilon}, \forall \epsilon, |\epsilon| > |\bar{\epsilon}| \Rightarrow SW^u > SW^a. \quad (31)$$

- *Uniform insurance Pareto dominates actuarial insurance inasmuch as the relative size η of population exposed to prevention externalities is important.*

$$\forall \epsilon, \exists \bar{\eta}, \forall \eta, \eta > \bar{\eta} \Rightarrow SW^u > SW^a. \quad (32)$$

Proof. See AppendixE. □

The comparative statics established by Proposition 2 can be summarized on Figure 3.

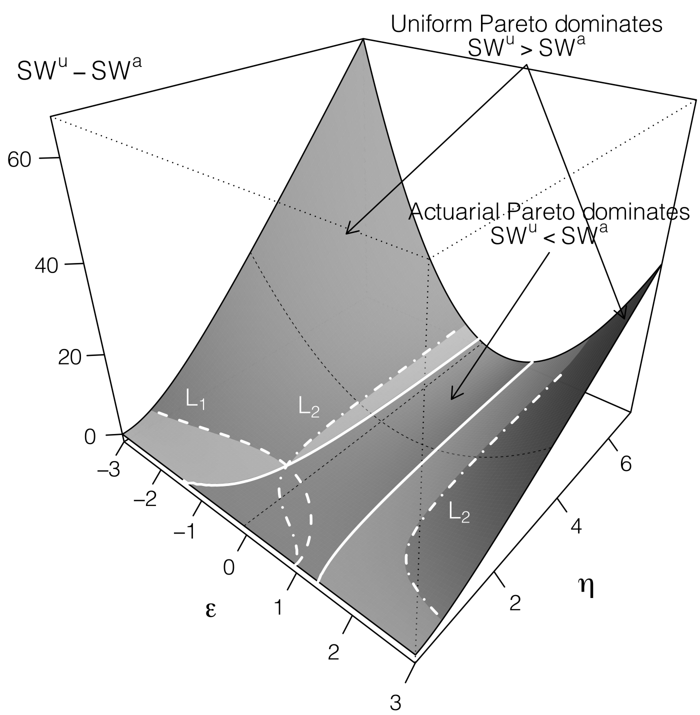

Figure 3: Comparative statics of the difference of social welfare between uniform and actuarial insurance ($p^0 LN_1/c_1 = 0.1$, $c_1 = c_2 = 10$)

Figure 3 compares the social welfare under uniform and actuarial insurance still in the case where $p^0LN_1/c_1 = 0.1$ and with equal cost coefficients.[19] The two white curves on Figure 3 correspond to the set of points (ϵ, η) where the social welfare is the same under uniform and actuarial insurance ($SW^u = SW^a$). Between these two white curves, that is when externalities have a small effect, actuarial insurance Pareto dominates uniform insurance ($SW^a > SW^u$). Outside these two white curves, that is when externalities have an important effect because either $|\epsilon|$ or η is large, uniform insurance Pareto dominates actuarial insurance ($SW^u > SW^a$). The three regions in shaded white correspond to the cases where Pareto dominance of uniform insurance requires transfers between the two jurisdictions.

Is uniform insurance likely to Pareto dominate actuarial insurance in practice? Comparative statics of the difference of jurisdictions' wealths between uniform and actuarial insurance have been illustrated in the case where $p^0LN_1/c_1 = 0.1$ and with equal cost coefficients. Assuming $p^0LN_1/c_1 = 0.1$ means that in autarky under actuarial insurance the optimal prevention effort by jurisdiction 1 corresponds to a risk reduction of 10% (Equation 19). The chosen cost coefficient (here $c_1 = c_2 = 10$) is a scale coefficient that only impacts the magnitude of wealth difference. This rough calibration of the model parameters enables to exhibit realistic cases where uniform insurance is likely to Pareto dominate actuarial insurance.

These results illustrate the general theory of second best formalized by Lipsey and Lancaster (1956-1957). In the absence of externalities, actuarial insurance corresponds to the first best policy. However, when the effect of externalities is important, uniform insurance leads to a larger welfare than actuarial insurance.

7. Concluding remarks

This paper provides a simple and original theoretical framework to address the issue of flood exposure. Indeed, flood risk presents two major specificities. First, flood collective prevention choices (dams, levees, retention basins) are observable and for most they exert positive or negative externalities on neighboring jurisdictions. Second, flood coverage is based de facto or de jure on uniform contribution in many countries.

Uniform coverage scheme is often criticized for providing no prevention incentives. In the absence of prevention externalities, Picard (2008) illustrates the equity-efficiency tradeoff between actuarial and uniform insurance against natural disasters: the author shows that actuarial insurance is efficient, but generates inequity, whereas uniform insurance is inefficient but ensures equity. We revisit this tradeoff in the presence of prevention externalities, which is particularly relevant for floods and mudslides. As uniform insurance enables a partial integration of prevention externalities by jurisdictions, we show that it is more efficient than actuarial insurance if and only if the effect of prevention externalities is important.

[19] The whole set of points (ϵ, η) considered on Figure 3 verify the conditions of validity under uniform and actuarial insurance (AppendixA).

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AppendixA. Conditions of validity

Following (6) and (7), the conditions of validity are

Centralization	Autarky	
Uniform or actuarial insurance	Uniform insurance	Actuarial insurance
$\frac{p^0LN_1}{c_1}(1+\eta\epsilon) \leq 1,$	$\frac{p^0LN_1}{c_1}\frac{1+\eta\epsilon}{1+\eta} \leq 1,$	$\frac{p^0LN_1}{c_1} \leq 1,$
$p^0LN_1\left(\frac{\eta}{c_2}+\frac{\epsilon(1+\eta\epsilon)}{c_1}\right) \leq 1.$	$p^0LN_1\left(\frac{1}{c_2}\frac{\eta}{1+\eta}+\frac{\epsilon}{c_1}\frac{1+\eta\epsilon}{1+\eta}\right) \leq 1.$	$p^0LN_1\left(\frac{\eta}{c_2}+\frac{\epsilon}{c_1}\right) \leq 1.$

AppendixB. Wealths in centralization without transfers between the two jurisdictions

In centralization and in the absence of transfers between the two jurisdictions, the wealth of each jurisdiction is respectively under uniform and actuarial insurance

$$W_1^{u**} = N_1I - p^0LN_1 + \frac{(p^0LN_1)^2}{2c_1}(1+\eta\epsilon)^2\frac{1-\eta}{1+\eta} + (p^0L)^2\frac{N_1N_2}{c_2}\frac{\eta}{1+\eta}, \tag{B.1}$$

$$W_2^{u**} = N_2I - p^0LN_2 + (p^0L)^2\frac{N_1N_2}{c_1}\frac{(1+\eta\epsilon)^2}{1+\eta} + \frac{(p^0LN_2)^2}{2c_2}\frac{\eta-1}{1+\eta}, \tag{B.2}$$

$$W_1^{a**} = N_1I - p^0LN_1 + \frac{(p^0LN_1)^2}{2c_1}(1-\eta^2\epsilon^2), \tag{B.3}$$

$$W_2^{a**} = N_2I - p^0LN_2 + (p^0L)^2\frac{N_1N_2}{c_1}\epsilon(1+\eta\epsilon) + \frac{(p^0LN_2)^2}{2c_2}. \tag{B.4}$$

AppendixC. Wealths in autarky

In autarky, the final wealths are respectively under uniform and actuarial insurance

$$W_1^{u*} = N_1I - p^0LN_1 + \frac{(p^0LN_1)^2}{2c_1}\left(\frac{1+\eta\epsilon}{1+\eta}\right)^2 + (p^0L)^2\frac{N_1N_2}{c_2}\left(\frac{\eta}{1+\eta}\right)^2, \tag{C.1}$$

$$W_2^{u*} = N_2I - p^0LN_2 + (p^0L)^2\frac{N_1N_2}{c_1}\left(\frac{1+\eta\epsilon}{1+\eta}\right)^2 + \frac{(p^0LN_2)^2}{2c_2}\left(\frac{\eta}{1+\eta}\right)^2, \tag{C.2}$$

$$W_1^{a*} = N_1I - p^0LN_1 + \frac{(p^0LN_1)^2}{2c_1}, \tag{C.3}$$

$$W_2^{a*} = N_2I - p^0LN_2 + (p^0L)^2\frac{N_1N_2}{c_1}\epsilon + \frac{(p^0LN_2)^2}{2c_2}. \tag{C.4}$$

AppendixD. Comparison of centralization without transfers between the two jurisdictions and autarky

Under uniform insurance, the wealth difference in each jurisdiction is:

$$W_1^{u**} - W_1^{u*} = (p^0 L N_1)^2 \left(\frac{\eta}{1+\eta}\right)^2 \left(-\frac{(1+\eta\epsilon)^2}{2c_1} + \frac{1}{c_2}\right) \gtreqless 0, \tag{D.1}$$

$$W_2^{u**} - W_2^{u*} = (p^0 L N_1)^2 \left(\frac{\eta}{1+\eta}\right)^2 \left(\frac{(1+\eta\epsilon)^2}{c_1} - \frac{1}{2c_2}\right) \gtreqless 0. \tag{D.2}$$

In the absence of externalities, the necessary and sufficient condition for $W_1^{u**} - W_1^{u*} \geq 0$ and $W_2^{u**} - W_2^{u*} \geq 0$ is $1/2 \leq c_1/c_2 \leq 2$.

Under actuarial insurance, the wealth difference in each jurisdiction is:

$$W_1^{a**} - W_1^{a*} = -\frac{(p^0 L N_1)^2}{2c_1}\eta^2\epsilon^2 \leq 0, \tag{D.3}$$

$$W_2^{a**} - W_2^{a*} = \frac{(p^0 L N_1)^2}{c_1}\eta^2\epsilon^2 \geq 0. \tag{D.4}$$

AppendixE. Proof of Proposition 2

By adding up (27) and (29), we get the difference of social welfare between uniform insurance and actuarial insurance:

$$SW^u - SW^a = W_1^{u*} - W_1^{a*} + W_2^{u*} - W_2^{a*}, \tag{E.1}$$

$$= \frac{(p^0 L N_1)^2}{c_1}\left(\frac{\eta}{1+\eta}\right)^2 \left(\eta\epsilon(\epsilon-1) + \frac{1}{2}\left(\epsilon^2 - 1 - \frac{c_1}{c_2}\right)\right). \tag{E.2}$$

$$SW^u - SW^a > 0 \Leftrightarrow \eta\epsilon(\epsilon-1) + \frac{1}{2}\left(\epsilon^2 - 1 - \frac{c_1}{c_2}\right) > 0, \tag{E.3}$$

$$\Leftrightarrow \eta > \bar{\eta} = \frac{1 + \frac{c_1}{c_2} - \epsilon^2}{2\epsilon(\epsilon-1)}, \tag{E.4}$$

$$\Leftrightarrow |\epsilon| > |\bar{\epsilon}| = \max\left\{\left|\frac{\eta + \sqrt{\eta^2 + (2\eta+1)(1+\frac{c_1}{c_2})}}{2\eta+1}\right|, \left|\frac{\eta - \sqrt{\eta^2 + (2\eta+1)(1+\frac{c_1}{c_2})}}{2\eta+1}\right|\right\}. \tag{E.5}$$