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Information Flows across the Futures Term Structure: Evidence from Crude Oil Prices

Delphine Lautier[†] Franck Raynaud[‡] Michel A. Robe[§]

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Abstract

We apply the concepts of conditional entropy, information transfers and directed graphs to investigate empirically the propagation of price fluctuations across a futures term structure. We focus on price relationships for North American crude oil futures because this key market experienced several structural changes between 2000 and 2014: financialization (starting in 2003), infrastructure limitations (in 2008-2011) and regulatory changes (in 2012-2014) in addition to big demand and supply shocks in the underlying asset market. We find large variations over time in the amount of information shared by contracts with different maturities. Although on average short-dated contracts (up to 6 months) emit more information than backdated ones, a dynamic analysis reveals that, after 2012, similar amounts of information flow backward and forward along the futures maturity curve. The mutual information share increased substantially starting in 2004 but fell back sharply in 2012-2014. In the crude oil space, our findings point to a puzzling re-segmentation of the futures market by maturity in 2012-2014. More broadly, they have implications for the Samuelson effect and raise questions about the causes of market segmentation.

Keywords: Information flows, Term structure, Entropy, Directed graphs, Futures, WTI

JEL Classification Codes: G10, G13, G14, Q49

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1 Introduction

Commodity futures market fulfill the key economic functions of allowing for hedging and price discovery. In these markets, two important questions arise.

First, are futures prices interconnected across the maturity curve? Theory suggests that they should be linked through the cost-of-carry relationship. In practice, however, such market integration requires cross-maturity arbitrage. Büyükşahin, Harris, Overdahl and Robe (2009) document that even the three largest U.S. commodity futures markets did not witness substantial activity in longer-dated derivatives until 2003 (crude oil) or later (corn and natural gas). This empirical reality suggests the possibility of changes in cross-maturity informational linkages in the past decade.

Second, assuming that different-maturity futures prices are interconnected, where in the term structure do information shocks originate, and which other parts of the term structure do they reach? Is the directionality of these information flows stable over time? Theoretically, the physical or spot market is the place for the absolute price to emerge as a function of the supply and demand for the underlying asset. In turn, the derivative market allows for relative pricing: futures prices derive from the spot price. Under normal circumstances, the information should thus flow from the underlying asset to the derivative instrument: this a central assumption in term structure models of commodity prices such as Schwartz's (1997). Yet, amid a massive increase in far-dated commodity futures trading after 2003, might one not expect to also observe an informational flow from the far end to the short (physical) end of the futures curve?

We apply the concepts of conditional entropy and directed graphs to answer these questions empirically. The New York Mercantile Exchange's (Nymex) West Texas Intermediate (WTI) sweet crude oil futures market provides an ideal setting for our analysis. Among all commodity markets, the WTI futures market boasts the highest level of activity together with the greatest number of far-out delivery dates (up to seven years). Graph theory gives us a way to describe all the prices we are examining as well as the links between them. Information theory allows for the study of the informational content of these prices.

A key reason underlying our choice of graph theory is that, insofar as all the futures prices that we are studying create a system, then this system is a complex one: it is made of many components that may interact in various ways through time. On any sample day year, there are more than 30

different delivery dates in the crude oil market, so we have over 870 pairs of maturities to examine after accounting for directionality. Moreover, such linkages may change through time as a result of evolving trading practices in the past 15 years. Finally, chances are few that the relationships between different maturities are always linear.

Graph theory can deal with such complex systems and generate statistical measures in order to examine them. It allows for identifying some recurrent behavior (i.e., emergent rules), assessing the robustness of the empirical findings and checking for possible pathological patterns. A graph (or network) gives a representation of pairwise relationships within a collection of discrete entities. Each point of the graph constitutes a node (or vertex). In this article, a node corresponds to the time series of prices returns of a futures contract. The links (or edges) of the graph can then be used in order to describe the relationships between the nodes. More precisely, the graph can be weighted in order to take into account the intensities and/or the directions of the connections. We do both on the basis of information theory.

There are several ways to enrich the information contained in a graph through its links. In finance, for example, the connections between the nodes can be related to the correlations of price returns or to the positions of market operators. Here, as we are concerned with the information content of the futures contracts, we use information theory in order to enrich the links of the graph in two ways: first, to determine the intensities of the links; second, to obtain their direction. To the best of our knowledge, such an application is unprecedented in studies of futures term structures and of commodity markets.

The theory of information, first proposed by Shannon (1948), aims at quantifying information. Mutual information is a key concept in this context. It gives a precise assessment of the information shared by two random variables – in our case, the daily returns on futures with different maturities. Thus the mutual information shared by pairs of futures contracts constitutes the intensities of the links of the graph. Of course, each futures maturity might constitute a potential source as well as a potential recipient of information. In order to disentangle these two possibilities, we introduce directionality. We do so by relying on the concept of conditional entropy. The entropy (or degree of uncertainty) of our random variable is conditioned by another random variable: in our case, the lagged value of another futures contract's price. Finally, being able to construct directed graphs gives us the possibility to sort among futures contracts between transmitter *vs.* receiver and to see

how this transmission rule evolves according to time or to market conditions.

On average across our 2000-2014 sample period, we find that the nearby contract emits more information than any other maturity and that short-dated contracts (maturities up to 6 months) emit more information than backdated ones – a pattern consistent with the typical functioning of futures market. A dynamic analysis, however, reveals that the amount of information flows originating in the far end of the curve increased as cross-maturity integration progressed. Nowadays, similar amounts of information flow from the near and far ends of the maturity curve but the directionality of information flows (from near- to far-dated contracts or vice-versa) is less stable.

We find substantial variations over time in the amount of information shared by futures contracts with different delivery dates. Intermediate-maturity contracts (6 months to 2 years) share relatively more common information from other contracts and, in turn, emit relatively less information to other maturities. For all contracts, the common share increased dramatically after 2003 (amid tight oil supply conditions and the onset of commodity markets' financialization) but fell back sharply in 2012 (to pre-2005 levels) and fell further in 2013 and 2014 (to pre-2002 levels).

Taken together, our term structure findings point to a puzzling re-segmentation of the futures market by maturity in 2012-2014. More broadly, they have implications for Samuelson's (1965) hypothesis regarding the term structure of futures volatilities and they raise questions about the causes of market segmentation.

The paper proceeds as follows. Section 2 summarizes our contribution to the literature. Section 3 outlines our methodology, which is based on conditional entropy and information transfers. Section 4 presents the data and our empirical results. Section 5 concludes.

2 Literature

The present paper contributes to three literatures: on term structures and market segmentation, on causality, and on the use of graph theory in the context of financial markets.

The theoretical literature on the term structure of futures prices for commodities in general, and crude oil in particular, includes many distinguished contributions such as those of Schwartz (1997), Routledge, Seppi and Spatt (2000), Casassus and Collin-Dufresne (2005), Casassus, Collin-Dufresne and Routledge (2007), Carlson, Khokher and Titman (2007), Kogan, Livdan and Yaron (2009), Liu

and Tang (2010) and Baker (2014). The models proposed in those papers, however, do not deal with the possibility that market frictions may prevent different parts of the price curve from moving in sync.

Questions related to the information contained in a term structure of prices and the possible implications of market imperfections for segmentation date back to the works of Culbertson (1957) and Modigliani and Sutch (1966) on "preferred habitats" in bond markets. Spurred in part by interest rate behaviors during the 2008-2011 financial crisis and the so-called Great Recession, the past ten years have seen a resurgence of theoretical and empirical work on segmentation. The latter is defined as a situation in which different parts of the price curve are disconnected from each other. Gürkaynak and Wright (2012), who review this still-growing literature, conclude that "the preferred habitat approach (has) value, especially at times of unusual financial market turmoil" (p. 360). For example, D'Amico and King (2013) document the existence of a "local supply" effect in the yield curve in 2009 during the U.S. Federal Reserve's unprecedented program to purchase $300 billion of U.S. Treasury securities.

Research on possible term structure segmentation in commodity futures markets deals almost exclusively with the crude oil market, which boasts the highest trading volumes and (in the United States) contract maturities extending up to seven years. In contrast to interest rate markets, prior work in the WTI futures space suggests that the Lehman crisis and its direct aftermath did not witness an increase in market segmentation. Granted, on the basis of the informational value of futures prices, Lautier (2005) finds cross-maturity segmentation during the 1990's. She argues, however, that this phenomenon had become less strong by the end of her sample period in 2002. Indeed, using recursive cointegration techniques, Büyükşahin *et al.* (2011) document that WTI cross-maturity linkages became statistically significant in 2003-2004 and remained so through at least May 2011 (the end of their sample period).[1]

We complement this prior work in several ways: we quantify the information shared by different contracts according to their maturity, assess the direction of the information flows between maturities, and document how these measures have evolved through time. Our analysis, based

[1]Using a comprehensive, trader-level dataset of end-of-day futures positions and trader type, Büyükşahin *et al.* document that this market development can be attributed to what has been dubbed the "financialization" of commodity markets (specifically, the increased market activity by commodity swap dealers, hedge funds and other financial traders). See Büyükşahin and Robe (2011, 2014) for further evidence regarding the financialization of energy (2011) and other (2014) commodity markets; see Cheng and Xion (2013) for a review of the financialization literature.

on different techniques, confirms the prior finding of increasing market integration until 2011. We show, however, that when discussing market segmentation one must distinguish between forward *vs.* backward flows of information, as both types of flows are not equally impacted by segmentation. Furthermore, in sharp contrast to the cross-maturity integration that characterized the 2004-2011 period, we show that different parts of the WTI term structure became much less integrated in 2012-2014.

Insofar as it focuses on price relationships, the present paper belongs to a vast literature on prices linkages. If the spatial dimension of market integration has been analyzed in depth elsewhere for equities and currencies as well as for commodities, cross-maturity linkages have not. Other than the two articles discussed above, prior work on information flows in the crude oil market abstracts from term structure issues and investigates instead the relationship between spot and futures prices.[2] In that context, a central question is whether price discovery takes place on the futures or the spot market (Garbade and Silber, 1983). While early studies tend to rely on Granger causality to provide an answer, a number of papers apply other techniques in an attempt to tease out causality when the relationship between prices might be non-linear.[3]

The methodological choices in the present paper are likewise motivated not only by concerns about possible non-linearities but also by the sheer size of the system we consider (870 daily pairs of maturities, accounting for directionality). A number of papers use information theory or graph theory to investigate information transfers, but none look at a system as large as a term structure – see, e.g., Haigh and Bessler (2004), Bryant, Bessler and Haigh (2006), Wang (2010), Lautier and Raynaud (2012), Dimpfl and Peter (2014) and references cited therein. Compared with this body of work, our paper utilizes another type of graph allowing for high dimensional analysis: we study all the possible connections between the different maturities of the North American crude oil market. Moreover, as our analysis relies on information theory, our methodology is "model free:" contrary to what is done with a Granger analysis, we make no assumption regarding the nature of the relationships under scrutiny – it does not matter if they are linear or not.

[2]Kawamoto and Hamori (2011) are an exception. These authors look at WTI futures contracts with maturities up to nine months. Their focus, however, is on market efficiency and unbiasedness.

[3]See, e.g., Silvapulle and Moosa (1999), Switzer and El-Khoury (2007), Alzahrani, Masih and Al-Titi (2014) and references cited in those papers

3 Methodology

In order to study the interdependence of, and the directionality between, price movements for different futures contracts, we rely on the theory of information based on the notion of entropy proposed by Shannon (1948). This Section first presents entropy measures. Next, it discusses the concept of "mutual information" that we use as a proxy for market integration. Within this framework, mutual information is a key concept that quantifies the dependency between two random variables. Unlike correlations, the mutual information measure captures non-linear relationships between variables. Finally, we describe measures of information transfers that allow us to study directionality between different contracts' price changes.

3.1 Mutual Information

Consider a random variable X and its corresponding probability distribution $p(x)$. The "entropy" $H(X)$ captures the degree of uncertainty of X and is defined as:

$$H(X) = -\sum_{x} p(x) \log p(x) \tag{1}$$

where $\sum_x$ is a sum over all the possible states of X. The value of H increases with the number of possible states and, for a given such number, is highest when X is uniformly distributed.

Now consider two variables X and Y with joint probability distribution $p(x, y)$. The remaining entropy of X if the values of Y are known is given by the "conditional entropy":

$$H(X|Y) = -\sum_{x,y} p(x, y) \log \frac{p(y)}{p(x, y)} \tag{2}$$

If the distribution $p(x|y)$ is known, then this conditional entropy can be written as:

$$H(X|Y) = -\sum_{x,y} p(x, y) \log p(x|y) \tag{3}$$

In turn, the entropy $H(X, Y)$ is the amount of information revealed by evaluating X and Y simultaneously. It amounts to the information that is revealed by first evaluating the value of either of these two variables (i.e., by computing either the entropy $H(X)$ or $H(Y)$ and then revealing

the value of the other variable given what is known about the value of the first (i.e., by adding, respectively, the conditional entropy $H(Y|X)$ or $H(X|Y)$). That is:

$$H(X,Y) = H(X|Y) + H(Y) = H(Y|X) + H(X) = H(Y,X) \tag{4}$$

The "mutual information" between X and Y is:

$$M(X,Y) = \sum_{x,y} p(x,y) \log \frac{p(x,y)}{p(x)p(y)} \tag{5}$$

It is also possible to express the mutual information of X and Y as:

$$\begin{aligned} M(X,Y) &= H(X) - H(X|Y) \\ &= H(X) + (H(Y) - H(X,Y)) \\ &= H(X,Y) - H(X|Y) - H(Y|X) \end{aligned}$$

Intuitively, the mutual information $M(X,Y)$ measures the information that X and Y share, i.e., the uncertainty reduction compared to the case where X and Y are independent. Formally, if the entropies $H(X)$ and $H(Y)$, the conditional entropies $H(X|Y)$ and $H(Y|X)$ and the joint entropies $H(X,Y)$ and $H(Y,X)$ are all known, then the mutual information can be seen as the reduction of entropy of one variable when the other is known.

Given the above definitions, the mutual information is "symmetric" if we can interchange X and Y. In that case, $M(X,Y)$ cannot detect influence of X to Y and *vice versa*.

3.2 Information Transfer

As we aim to characterize each futures contract as a potential source or recipient of information, we adopt the formalism proposed by Schreiber (2000) to assess directionality to the flows of information between futures contracts. Suppose that the value of X at time $t+1$ depends on the value of Y at time t. Then, the "entropy rate" h_1 can be defined as:

$$h_1 = -\sum p(x_{t+1}, x_t, y_t) \log p(x_{t+1}|x_t, y_t) \tag{6}$$

If, to the contrary, X at time $t+1$ does not depend on Y at time t, then the entropy rate h_2 is:

$$h_2 = -\sum p(x_{t+1}, x_t, y_t) \log p(x_{t+1}|x_t) \tag{7}$$

The "entropy transfer" from Y to X is simply the difference between these two rates of entropy:

$$T_{Y\to X} = h_2 - h_1 = \sum p(x_{t+1}, x_t, y_t) \log \frac{p(x_{t+1}|x_t, y_t)}{p(x_{t+1}|x_t)} \tag{8}$$

The transfer from X to Y can similarly be written as:

$$T_{X\to Y} = h_1 - h_2 = \sum p(y_{t+1}, y_t, x_t) \log \frac{p(y_{t+1}|y_t, x_t)}{p(y_{t+1}|y_t)} \tag{9}$$

Using the definitions of conditional entropies, the two transfers can be rewritten as:

$$\begin{aligned} T_{Y\to X} &= H(X_{t+1}|X_t) - H(X_{t+1}|X_t, Y_t) \\ T_{X\to Y} &= H(Y_{t+1}|Y_t) - H(Y_{t+1}|Y_t, X_t) \end{aligned}$$

In the case of a linear dependency between two Gaussian random variables X and Y, the transfer entropy causality measure is equivalent to Granger causality (Barnett, Barrett and Seth, 2009). Granger causality is a statistical notion of predictive causality *via* vector auto-regression. Transfer entropy has the advantages of being model-free and of holding in the case of non-linearity.

3.3 Measurements

Equipped with the above definitions, one can construct several measures in order to quantify the properties of prices fluctuations in terms of information content and to differentiate, among futures contracts, between transmitter and receiver: the total amount of information sent from a maturity, the flows of backward and forward information and a directionality index.

Using Equation (9), we can compute the total amount of information sent from the maturity i to all other maturities $j \neq i$ as:

$$T_{i_S} =< T_{i\to j} >_{j, j\neq i} \tag{10}$$

and the information received by the maturity i is:

$$T_{i_R} =< T_{i \leftarrow j} >_{j, j \neq i} \tag{11}$$

where $<>_{j,j\neq i}$ denotes the average over all the contract maturities other than i.

We also compute the flows of forward and backward information to investigate changes in the direction of information. We use the notions of "forward flow" to capture the information emitted by short-term maturities in the direction of long-term maturities and of "backward flow" to capture the opposite. The forward flow ϕ_f is given by:

$$\phi_f = \sum_{X<Y} T_{X \to Y} \tag{12}$$

while the backward flow ϕ_b is given by:

$$\phi_b = \sum_{X>Y} T_{X \to Y} \tag{13}$$

In order to investigate through graph theory the properties of the directionality between two contracts X and Y, we construct an index of directionality by combining Equations (8) and (9):

$$D_{XY} = \frac{T_{X \to Y} - T_{Y \to X}}{T_{X \to Y} + T_{Y \to X}} \tag{14}$$

D_{XY} gives the *strength* of the directionality. It is bounded by -1 and 1. If D_{XY} is greater than 0, then the information flows from X to Y; If $D_{XY} < 0$, then the information flows from Y to X.

Computing D_{XY} for the entire sample period generates the matrix of directionality $\bar{D}_{XY}$ that represents the static full directed graph. By computing the index in two-year rolling windows we can get, at time t, the instantaneous directionality matrix $D_{XY}(t)$ and track its evolution over time.

Finally, we measure the degree of stability of the information in the graph by computing the survival ratio $\bar{S}_R(t)$ as the number of element of same sign in $\frac{1}{N} D_{XY}(t) \cap \bar{D}_{XY}$. If $\bar{S}_R(t) = 1$, then the system has the same pattern of information flows as in the static case, i.e., the market is stable.

At the other extreme, if $\bar{S}_R(t) = 0$, then the set of directed links has been completely rearranged – indicating disturbances in the flow of information.

4 Empirical Results

After a brief description of our dataset, we present empirical evidence on the information shared by futures contracts of different maturities and its evolution through time. Next, we introduce directionality and examine information transfers. Finally, we study the stability of these transfers.

4.1 Data

Our dataset consists of the daily settlement prices for Nymex's WTI light, sweet crude oil futures contracts from the 21^{sh} of January 2000 to the 25th of February 2014. We construct 33 time series of futures prices. The first 28 are for the 28 shortest-dated contract maturities (i.e., contracts with 1 to 28 months until expiration). The last five time series correspond, respectively, to contract maturities of 30, 36, 48, 60 or 72 months. Futures roll dates are calendar-based.[4]

Our empirical analyses use daily futures returns. We compute daily returns on contract i, r_i, as the logarithm price difference: $r_i = (\ln F_i(t) - \ln F_i(t - \Delta t))/\Delta t$, where $F_i(t)$ is the price of the futures contract with maturity i at t and Δt is the time interval between consecutive sample days.

Figure 1 depicts the evolution of WTI futures prices and returns in our sample period. For readability, we focus on the nearby, one- and two-year contracts. The *right* panel of the figure shows that the realized returns' volatility is lower for the two backdated contracts than for the nearby futures, an empirical fact consistent with the Samuleson (1965) hypothesis that volatility should increase as a futures contract's maturity nears.[5]

Quite obvious in the *left* panel of Figure 1 are the sharp oil price rise in 2007-2008 and the consequent precipitous price decrease after August 2008. Equally notable, and especially relevant to the present study, is the difference in the relative behaviors of nearby *vs.* longer-dated contracts

[4] That is, we use the Nymex calendar to determine which contract has (for example) a one- *vs.* two-month maturity and when the first-deferred contract becomes the nearby contract.

[5] We obtain the same volatility ranking with futures rolled based on the preponderance of the open interest rather than calendar dates. Bessembinder, Coughenour, Seguin and Monroe Smoller (1996) give an elegant theoretical analysis of conditions under which the Samuelson (1965) effect holds, such as asset markets in which spot price changes include a temporary component (so that investors expect mean-reversion) or, alternatively, the assumption that information revelation about spot prices is systematically clustered around futures expiration dates (as in Anderson and Danthine, 1983). For more recent empirical evidence on the prevalence of the Samuelson effect, see Brooks (2012).

at the beginning (before 2004) and at the end (2012-2014) of our sample. As previously documented in Lautier (2005) for pre-2002 data and Büyükşahin *et al.* (2011) for the May 2000 to May 2011 period, Figure 1 shows that the one-year (*two-year*) futures prices did not move in sync with the nearby price prior to 2003 (*2004*) but started doing so soon thereafter. Figure 1 extends these prior empirical findings by showing that, starting in late 2011 and accelerating in 2012, a disconnect has reappeared between short- and further-dated WTI futures.

4.2 Mutual Information: A Proxy for Market Integration

Changes in the WTI market during the period under consideration can be characterized more formally and thoroughly through the lens of mutual information. Recall that the source of information, in our case, is prices fluctuations. Thus, the mutual information measure captures the simultaneous dependency or synchronous moves in prices.

4.2.1 Mutual information shared by all contract maturities

Figure 2 depicts the global dynamic behavior of the total mutual information in the system. Precisely, it plots the evolution over time of the mutual information shared by all maturities, $M^T(t)$:

$$M^T(t) =< M_{ij}(t) >_{i,i>j} \tag{15}$$

where the $M_{ij}(t)$ are the elements of the matrix of mutual information computed on day t using daily returns for the prior two years (500 trading days) and $<>_{i,i>j}$ denotes the averaging operator over the relevant contract maturities i.

The high values of M^T between 2004 and the beginning of 2011 are evidence of strong cross-maturity linkages.[6] As an increase in M^T can be interpreted in terms of greater integration of the futures market for crude oil during that period, this finding complements the cointegration-based study of this phenomemon by Büyükşahin *et al.* (2011).

Figure 2, however, also shows that the mutual information M^T decreases sharply after 2011 and

[6] To provide a visual reference for the statistical significance of the measure, we generate a benchmark by "shuffling" (i.e., using permutations of) the time index of each analyzed dataset. The resulting "shuffled" time series have the same statistical properties (mean, variance and higher moments) as the original ones but temporal relationships are removed. The resulting benchmark (i.e., the "shuffled" mutual information) is plotted in red in Figure 2. Unlike the actual series ($M^T(t)$, left-hand scale in black), the counterfactual (right-hand scale) is close to 0 and does not display any systematic pattern over time.

drops further, to a very low level, in 2013-2014. This finding is novel. It provides formal support for our interpretation of Figure 1 in the previous subsection: starting in 2012, different parts of the term structure of WTI futures prices have become much less integrated.

This apparent re-segmentation of the WTI market across futures delivery dates follows a partial geographic segmentation of world crude oil markets after the Summer of 2008 as evidenced by the emergence of a large differential between the prices of North American (WTI) and world (Brent) benchmark crudes. This large Brent-WTI price differential has been linked by Fattouh (2010), Pirrong (2010) and Büyükşahin *et al.* (2013) to two developments in the physical market for crude oil: infrastructure constraints at the delivery point for WTI futures in Cushing, OK, and a divergence in supply-side conditions in North America *vs.* the rest of the world. In contrast, rather than physical-oil market fundamentals, Büyükşahin *et al.* (2011) show that the WTI futures market's cross-maturity integration in 2004-2011 stemmed from a large increase in (cross-maturity) spread trading by hedge funds and other financial institutions amid what has been dubbed the "financialization of commodities." Intriguingly, the 2012-2014 re-segmentation evident in Figure 2 itself has been taking place amid a massive growth in North American oil production, export restrictions and transportation constraints, as well as new derivatives regulations in the United States. Correlation, though, need not imply causation. Our findings therefore raise the important question of the respective roles played by fundamentals *vs.* trading activity and (dis)incentives in the WTI market's re-segmentation.

4.2.2 Mutual information for each contract maturity

Figure 3 gives more insight into the developments identified in Figure 2 by depicting the mutual information for *each* contract maturity over the course of our sample period. For each maturity i and each day of the sample period t, the level of mutual information $M_i(t) = < M_{ij}(t) >_{j,j\neq i}$ is plotted in a color ranging from blue (very low) to green, yellow, orange or red (very high).

Figure 3 shows that, for all maturities, the mutual information is much higher in $2004 - 2011$ than at other times. Figure 3 also provides evidence that all futures prices do not have the same informational content. To wit, at each point in time, the graph "temperature" is typically cooler at the short end of the term structure (situated on the top of the plot) than at the far end of the curve (on the bottom of the graph), showing that near-dated contracts usually contain less mutual

information. This finding is consistent with the notion that short-dated crude oil futures prices are more volatile.[7] In contrast, there is a lot of mutual information for middle maturities: at any given time, the two extremities of the futures maturity curve share less information with the others than the intermediate one. Overall, these results suggest that the WTI futures term structure consists of three main segments: from the first to the third months, from the 4th to the 30th months, and finally the furthest-out delivery dates (those beyond the 30th month).

An important development took place in $2012 - 2014$, a period when the graph temperature becomes much cooler across the entire spectrum and is coolest for backdated contracts (those with maturities greater than three years). Figure 3 shows that the middle part of the maturity curve, where the amount of mutual information is the highest, is also larger in 2004-2011 than before or after. In other words, Figure 3 establishes that the integration phenomenon observed in Figure 2 comes principally from what happens at intermediate maturities.

Figure 4 provides additional evidence of this fact by averaging, over the sample period period, the information that a specific maturity shares with all others. Each point on the curve gives the average mutual information for each maturity. The bars around each point show the average variance of the measure over the sample period.

Figure 4 shows that the average mutual information is a hump-shaped function of contract maturity. It reaches its maximum near the 18 months maturity. Again, there is more mutual information at the back end of the curve (which we capture up to 6 years out) than at the front end (up to 3 months out). Whereas the shortest maturities are mostly influenced by shocks emerging in the physical market, the behavior of the longest maturities might be related to other factors such as expectations of future supply and demand for the commodity, technological changes, future discoveries, or possibly a lack of liquidity in the WTI futures markets.[8]

4.3 Information flows between maturities

A key component of the analysis of cross-maturity linkages is the examination of information transfers between different maturities. To answer the question of which side of the term structure is the transmitter and which one is the receiver, we first perform a static analysis across the whole sample

[7] See, e.g., Robe and Wallen (2014) for evidence on the term structure of WTI implied volatilities in 2000-2014.
[8] For a discussion of these likely explanatory factors, see e.g. Cortazar and Schwartz (2003).

period and then carry out a dynamic analysis using rolling windows of two years (500 trading days).

4.3.1 Static analysis

We start by computing information transfers over the entire sample period. This approach give us a picture of the "average" behavior of the system, i.e., of the information sent globally from the short term and received by the long term and *vice-versa*.

Figure 5 depicts the informational transfer between maturities recorded on the whole period. The black line corresponds to the information emitted by each maturity, given by equation 10; the red one shows the information received by each contract, computed using equation 11. The bars represent, for each maturity, the average variance recorded for the measure; they are particularly large for the information received on the long-term maturities.

Figure 5 shows that maturities up to one and one half years (preciseley up to and including the 19-month contract) emit more than they receive. Most of the information is sent to the far-out maturities. Once contract maturities extend beyond 6 months, the information emitted decreases with the maturity although the pattern levels off beyond the two-year mark. The information received exhibits a different pattern: it is high for the first three maturities, lowest for maturities ranging from 6 to 18 months and highest for maturities of 27 months and beyond (with the maximum value reached at the back end of the term structure). Intuitively, these static results imply that market participants whose "preferred habitat" (Modigliani and Sutch, 1966) is the back end of the maturity curve are more likely to be the object of a shock than to be the source of one.

In sum, the dependence structure between maturities depends on whether one considers either backward or forward information flows. These results therefore establish that market segmentation is not the same for the information emitted and for the information received.

4.3.2 Dynamic analysis

A natural question is whether the pattern of information transfers has changed over time. Figures 6, 7 and 8 provide the answer.

Figures 6 and 7 provide a dynamic analysis of, respectively, the information sent or received by each maturity to or from all other maturities. On each day from February 2002 to February 2014, the values are computed using, respectively, Equation 10 or 11 and daily returns in the prior two

years (500 trading days).

These two figures identify four periods: 2000 to 2004, 2005 to the Summer of 2008, Fall 2008 to 2011, and $2012-2014$. Through 2011, these two figures broadly confirm the pattern observed in the static analysis of Figure 5: the front end of the curve sends out more information than it receives, while the reverse is true for the back end. Figure 6 (*Figure 7*), however, shows that the information sent (*received*) by short-dated (*long-dated*) contracts is generally lower after 2004. Figure 7, in particular, shows medium- and far-dated contracts receiving very little information from other parts of the term structure during the period that started with Lehman Brothers' demise in September 2008 and ended with the European debt crisis of 2011. The most striking development uncovered by the dynamic analysis is seen in $2012-2014$. In that period, whereas backdated contracts (those with maturities of three years or more) again receive substantial amounts of information from other parts of the term structure, contracts with maturities of two to three years do not: instead, they send out at least as much information to other maturities as do near-dated contracts.

Figures 8 provides another lens through which we can evaluate these developments. Unlike Figures 5 to 7, Figure 8 does not distinguish between different maturities: instead, it illustrates what is emitted by all maturities (forward flows, Equation 12) and what is received by all of them (backward flows, Equation 13) at various points in time during our sample period. On each day from February 2002 to February 2014, the values of ϕ_f and ϕ_b are computed using daily futures returns for the prior two years (500 trading days).

Figure 8 highlights big changes. Both forward and backward flows decrease from 2000 to 2009 (backward flow) or 2010 (forward flow). This pattern reflects the progressive cross-maturity integration of the WTI futures market in the last decade. In $2012-2014$, however, both flows increase.[9]

In $2000-2009$, the forward flow is almost always much stronger than the backward flow. Thereafter, however, the amplitude of the two flows is comparable. Put differently, whereas in the beginning of our sample period, the term structure of futures prices was generally more prone

[9] To provide a visual benchmark for the statistical significance of the changes depicted by the red and black curves, we generate benchmark counterfactual forward and backward flow measures, $\phi_{f,shuffle}$ and $\phi_{b,shuffle}$, by permutating ("shuffling") the time index of each analyzed dataset. The resulting time series have the same statistical properties (mean, variance and higher moments) as the original ones but temporal relationships are removed. The resulting information transfers are shown in green (forward flow $\phi_{f,shuffle}$) and blue (backward flow $\phi_{b,shuffle}$). Unlike the actual forward and backward flows (depicted in black and red, respectively), the counterfactual flows fluctuate very little. Crucially, $\phi(t)_{f,shuffle}$ and $\phi(t)_{b,shuffle}$ are almost equal for all t: the counterfactual directionality measure computed with equation 14 is therefore close to 0.

to influence from shocks arising at the near end of the maturity curve, this is not true after 2010: the short-term prices (and, hence, physical prices) can be influenced by price fluctuations moving backward from far-dated contracts. In other words, the driving forces of price movements seem to have become comparable all along the term structure, and information nowadays propagates as easily in the forward direction as in the backward direction.

These findings raise questions regarding the so-called Samuelson effect. Samuelson (1965) hypothesized that futures prices volatility should increase as futures contracts approach their maturity. In theoretical models of the Samuelson effect, such as Deaton and Laroque (1992, 1996), the volatility of futures prices stems from shocks that arise in the physical market and are transmitted to the paper market. The term structure of volatilities is therefore downward-sloping: the direction for the propagation of shocks is forward (i.e. from short to long-term maturities) with a progressive absorption as contract maturity increase. However, our empirical results show that, in the later part of our sample period, there are price shocks coming from the far end of the futures term structure that spread to shorter maturities (and hence, arguably, to the physical market). In other words, it is nowadays possible to have backward propagation of prices shocks from the far end of the prices curve to the physical market.

4.3.3 Properties of information transfer

We rely on graph theory to more formally assess the stability of the information transfers. Precisely, we build a directed graph and we examine its stability. The nodes of the graph stand for time series of futures price returns, *i.e.*, one node per maturity. The graph's links are oriented according to the matrix of directionality D_{XY}, which measures the strength of the information transfer: for a couple of nodes (X,Y), if the element of the matrix $T_{X\to Y}$ is greater than $T_{Y\to X}$, then the edge is oriented from X to Y, otherwise from Y to X.

Figure 9 presents the directed graph extracted from the directionality matrix computed in the static case (i.e., for the entire $2000-2014$ sample period). It shows that the far-out maturities, which for readability we have positioned at the center of the graph, are those to which the links point. This graph provides us with a benchmark case, not only in that it represents what happens over the entire sample period but also in that the empirical results respresented by Figure 9 appear to support the conventional view of how a futures markets operates (specifically that prices shocks

are thought to form in the physical market, here represented by the short maturities, and transmit to the paper market, here made up of further-out maturities).

More information is given by a connectivity analysis, which we summarize in Figure 10. This Figure shows the fraction of all outer (i.e., outgoing) links associated with each maturity. The first 12 maturities are each associated with the most other maturities, i.e., the first year of the term structure sends the most information. Furthermore, the 18 first maturities send information to more than 50 percent of all other WTI maturities. In contrast, the deferred maturities receive a lot of information and send information to well under half of the other maturities.

A natural question is the stability of those relationships. Figure 10 provides the answer by plotting the variance of the outer links as a fraction of all links. The red line in that figure shows that the variance is much higher for the near-dated (first three months) and far-dated (more than three years) maturities, whereas the fraction of all outer (i.e., outgoing) links associated with middle maturities (six months to two years) is much less variable. Figure 11 provides further insights into the stability properties of the directed graph in Figure 9 by plotting the survival ratio $\bar{S}_R(t)$. The quantity $\bar{S}_R(t)$ is computed as the number of element of same sign in $\frac{1}{N} D_{XY}(t) \cap \bar{D}_{XY}$. If $\bar{S}_R(t) = 1$, then the system has the same flow of information as in the static case, i.e., the market is stable. If $\bar{S}_R(t) = 0$, then the set of directed links has been completely rearranged – indicating disturbances in the flow of information.

Figure 11 shows that, from 2000 until the end of 2010 and with the exception of a six-month period at the end of 2005, the survival ratio is generally higher than 0.7. This results indicates that most of the links remain in the same state as in the benchmark case. Thereafter, the ratio displays some variations but generally decreases – a finding suggesting a less stable market in recent years.

5 Conclusion

We apply the notions of conditional entropy and information transfers to empirically investigate the nature of price relationships across a futures term structure. The Nymex's WTI crude oil futures market, a large market that experienced a number of participatory and regulatory changes between February 2000 and February 2014, provides an ideal setting for our analysis.

We find substantial variations over time in the amount of information shared by futures with

different delivery dates. The common share increased dramatically starting in 2004 (amid tight oil supply conditions and the onset of commodity markets' financialization) but fell back sharply in 2012 (to pre-2005 levels) and fell further in 2013 and 2014 (to pre-2002 levels). On average over the entire sample period, short-dated contracts (maturities up to 3 months) emit more information than longer-dated ones. While this pattern seems consistent with the typical functioning of futures market, a dynamic analysis using rolling windows reveals that the information flows originating at the back end of the maturity curve have increased over time. Nowadays, similar amounts of information flow from both ends of the curve. Notably, the directionality of information flows (from near- to far-dated contracts or vice-versa) is less stable after 2005. These results suggest two natural venues for further research.

First, our term structure findings raise a theoretical question regarding the Samuelson (1965) hypothesis that a futures contract price's volatility should be inversely related with the contract's time to maturity. In theoretical models of the Samuelson effect, such as Deaton and Laroque (1992, 1996), the volatility of futures prices stems from shocks that arise in the physical market and are transmitted to the paper market. The term structure of volatilities is therefore downward-sloping: the direction for the propagation of shocks is the forward direction, i.e. from short to long-term maturities, with a progressive absorption as the maturity rises. However, our empirical results show that, in the second part of our sample period, there are prices shocks coming from the far end of the futures term structure that spread to the shorter maturities – and thus, arguably, to the physical market. In other words, it is now possible to have backward propagation of prices shocks, from the far end of the prices curve to the physical market. A theoretical model is needed to determine whether physical fundamentals or paper market conditions may be responsible for such a pattern of information flows in a term structure.

Second, our analysis establishes that the WTI market was segmented until 2003, integrated between 2004 and 2011, and segmented once more in 2012-2014. Questions regarding the information provided by a term structures of futures prices and the possible presence of a market segmentation affecting different parts of this term structure date back to Modigliani and Sutch (1966). In the WTI market that is the subject of our analysis, Büyükşahin *et al.* (2011) document that segmentation between the near, middle and far ends of the futures maturity curve, which had prevailed prior to 2000 (Lautier, 2005), all but disappeared after 2003. Those authors, using data from June 2000 to

May 2011, also show that cross-maturity market integration survived the 2007-2008 commodity price boom and crash, the Lehman crisis and the Great Recession. In the present paper we document, however, that the WTI market has since 2011 once again become segmented – a calamity for market participants who use backdated futures to hedge long-term price risk. The analysis of Büyükşahin *et al.* (2011) suggests that the unprecedented WTI market integration across the term structure in 2003-2011 was due to a combination of tight oil supply conditions and the onset of commodity markets' financialization. One therefore wonders if, in a similar vein, physical-market developments or a pullback by financial insitutions from participating at the far end of the WTI futures term structure explains the post-2011 re-segmentation of the market. Answering this question requires access to non-public trader-level trading data: we therefore leave it for further research.

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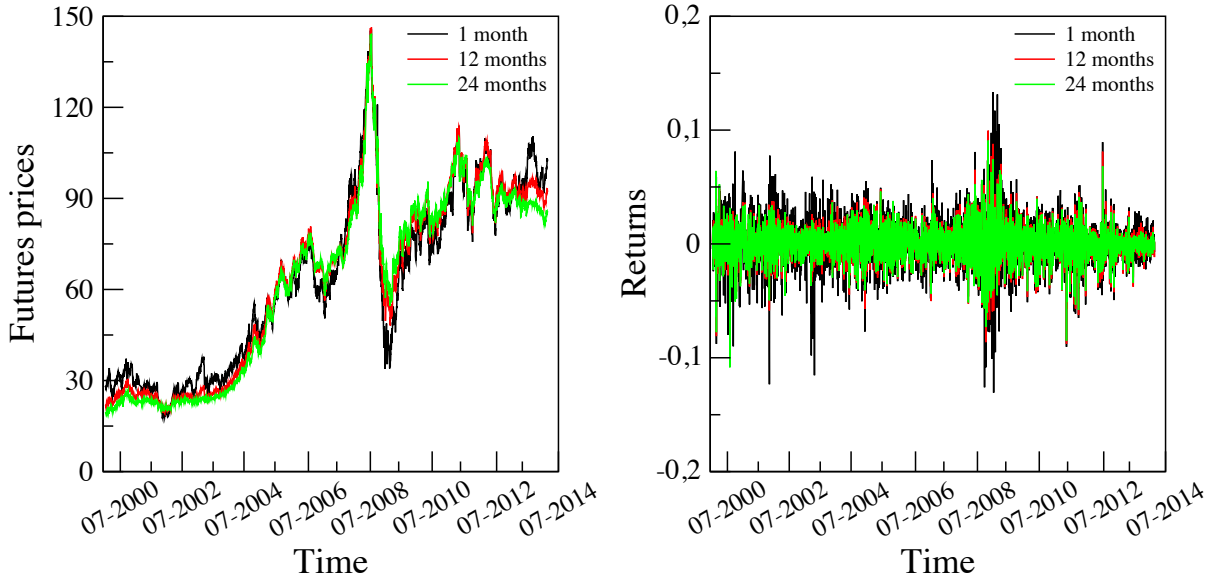

Figure 1: WTI crude oil futures prices and returns, 2000-2014

Note: Figure 1 depicts the evolution of the nearby, one- and two-year out WTI crude oil futures prices and returns in our sample period (January 2000 to February 2014; Source: Nymex). Prices are Nymex end-of-day settlement values. Futures roll dates are calendar-based. Daily futures returns are computed as the daily logarithm price differential r_i, with: $r_i = (\ln F_i(t) - \ln F_i(t - \Delta t))/\Delta t$, where $F_i(t)$ is the price of the futures contract with maturity i at time t and Δt is the time interval between two consecutive sample days.

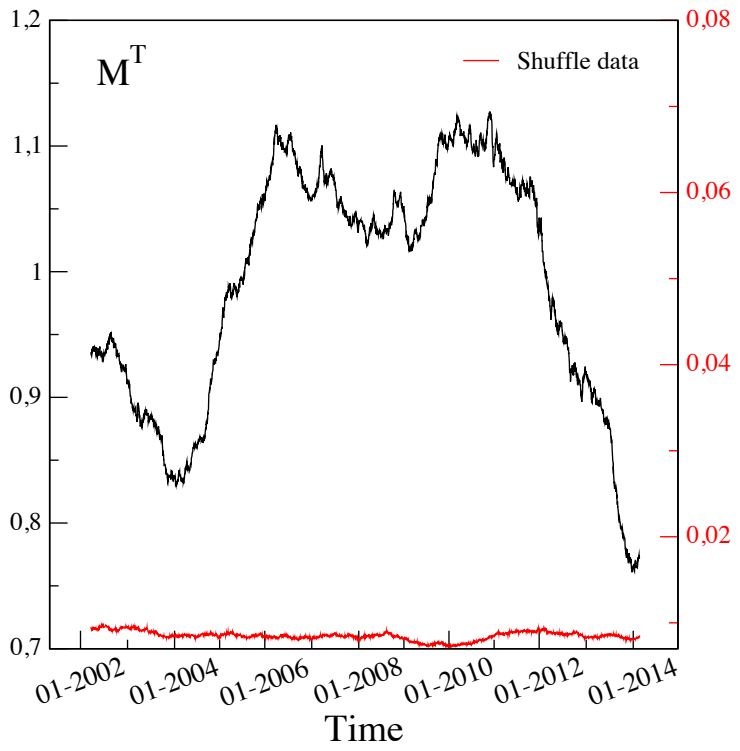

Figure 2: Mutual information shared by all maturities, 2000-2014

Note: Figure 2 plots the evolution over time of the mutual information shared by all WTI futures contract maturities, $M^T(t) =< M_{ij}(t) >_{i,i>j}$ where the $M_{ij}(t)$ are the elements of the matrix of mutual information computed on day t using daily returns for the prior two years (500 trading days) and $<>_i$ denotes the averaging operator over the relevant contract maturities i. The increase of M^T from 2004 until the end of 2010 shows that cross-maturity linkages are becoming more and more intense and can be interpreted as a higher integration of the futures market for crude oil. To provide a visual reference for the statistical significance of the changes depicted by the black curve, we generate a benchmark by "shuffling" (i.e., using permutations of) the time index of each analyzed dataset. The resulting "shuffled" time series have the same statistical properties (mean, variance and higher moments) as the original ones but temporal relationships are removed. The resulting benchmark (i.e., the "shuffled" mutual information) is plotted in red. Unlike the actual series ($M^T(t)$, left-hand scale in black), the counterfactual (right-hand scale in red) is close to 0 and does not display any systematic pattern over time.

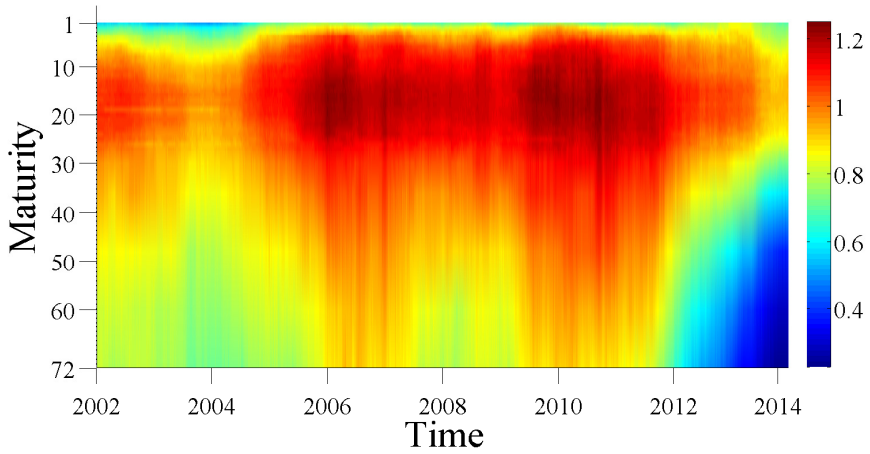

Figure 3: Mutual information for each contract maturity, 2000-2014

Note: Figure 3 depicts the mutual information for futures maturity over the course of our 2000-2014 sample period. For every maturity $i = 1, 2, ..., 72$ and every day between February 2002 and February 2014, the level of mutual information is computed using daily returns of the previous 500 trading days (two years) and is displayed in a color ranging from blue (very low) to green, yellow, orange or red (very high).

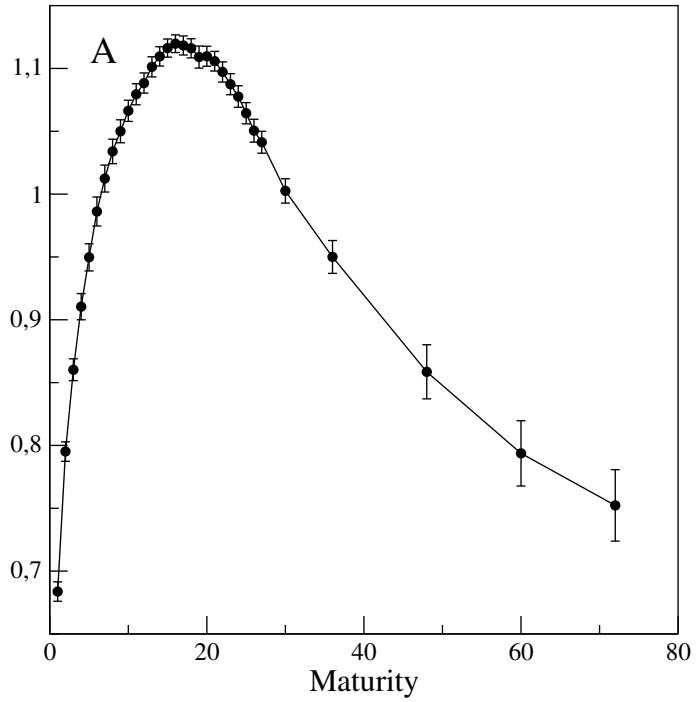

Figure 4: Average mutual information per maturity, 2000-2014

Note: Figure 4 shows the average A, computed over the entire 2000-2014 sample period period, of the information that a specific maturity shares with all others. Each point on the curve gives the average mutual information for each maturity. The bars around each point show the average variance of the measure over the whole sample period.

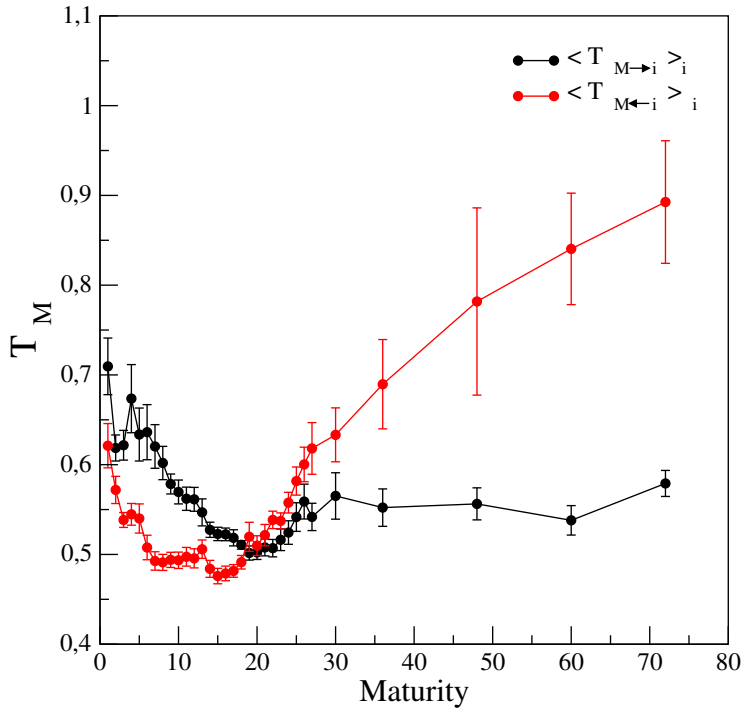

Figure 5: Average information transfer between maturities, 2000-2014

Note: Figure 5 depicts the informational transfer between maturities for our entire 2000-2014 sample period. The black line corresponds to the information emitted by each maturity, given by Equation 10; the red line shows the information received by each contract, computed using Equation 11. The bars represent, for each maturity, the average variance recorded for the measure.

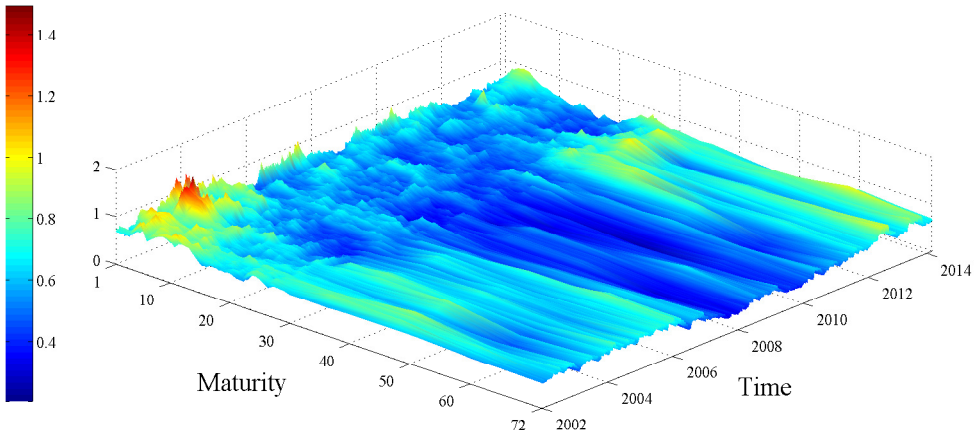

Figure 6: Information sent from each maturity to all others, 2002-2014

Note: Figure 6 shows the information sent by each maturity to all other maturities at each point in time during our sample period. On each day from February 2002 to February 2014, the values are computed using Equation 10 and daily returns in the prior two years (500 trading days).

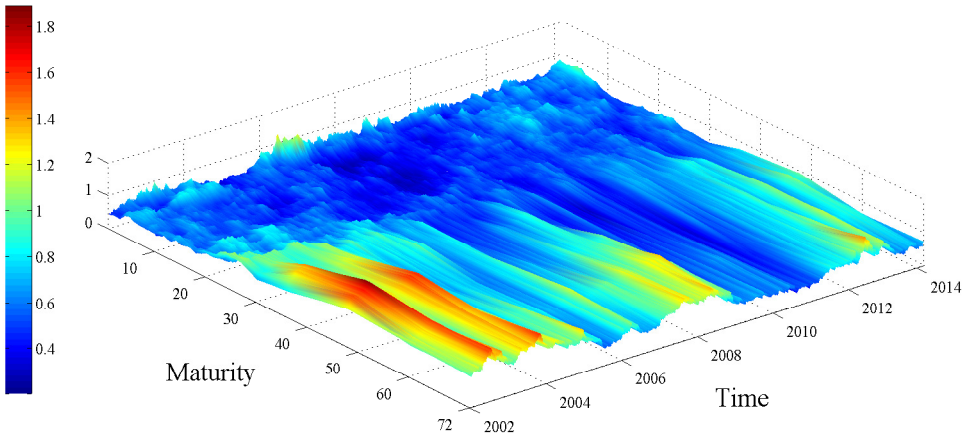

Figure 7: Information received by each maturity from all others, 2002-2014

Note: Figure 7 shows the information received by each maturity to all other maturities at each point in time during our sample period. On each day from February 2002 to February 2014, the values are computed using Equation 11 and daily returns in the prior two years (500 trading days).

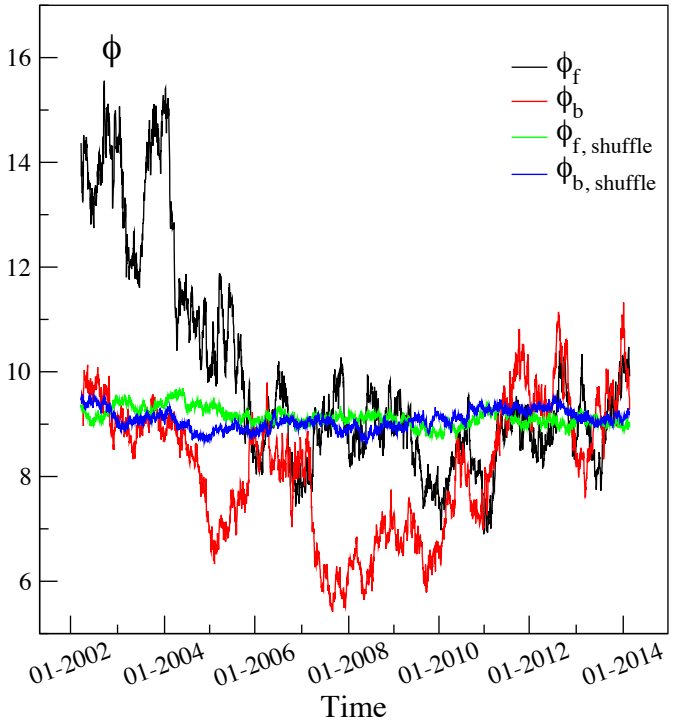

Figure 8: Information transfer between maturities, 2000-2014

Note: Figure 8 illustrates what is emitted by all maturities (forward flows ϕ_f, black curve) and what is received by all of them (backward flows ϕ_b, red curve) at various points in time. On each day from February 2002 to February 2014, we compute the values of ϕ_f and ϕ_b applying, respectively, Equations 12 or 13 to daily futures returns for the prior two years (500 trading days). To provide a visual benchmark for the statistical significance of the changes depicted by the red and black curves, we generate two benchmarks, $\phi_{f,shuffle}$ and $\phi_{b,shuffle}$, by "shuffling" (i.e., using permutations of) the time index of each analyzed dataset. The resulting "shuffled" time series have the same statistical properties (mean, variance and higher moments) as the original ones but temporal relationships are removed. The resulting benchmarks (i.e., the "shuffled" information transfers) are plotted in green (forward flow $\phi_{f,shuffle}$) and blue (backward flow $\phi_{b,shuffle}$). Unlike the actual forward and backward flows, the counterfactual flows fluctuate very little and are very close (i.e., directionality computed with equation 14 will be close to 0).

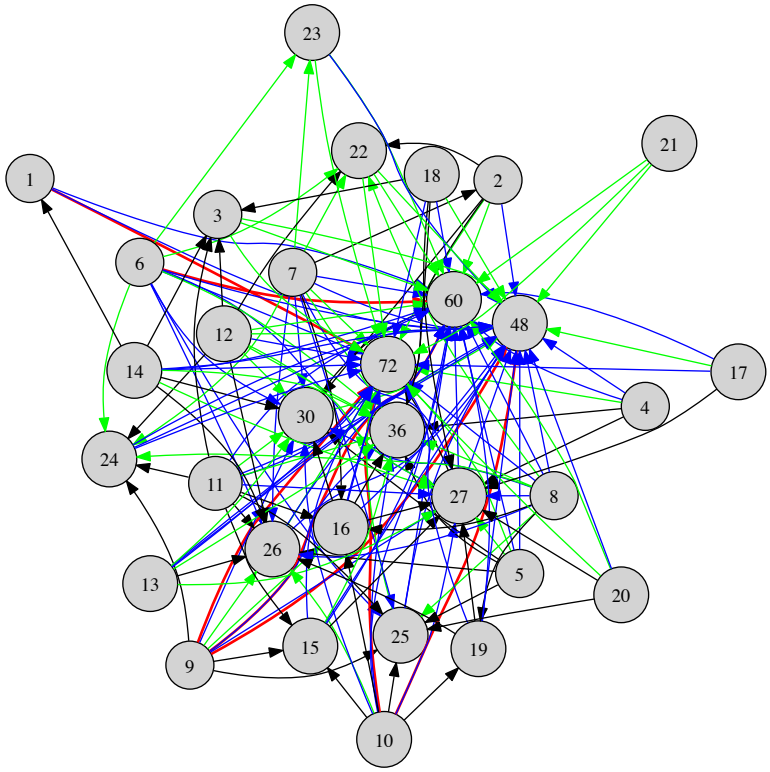

Figure 9: Full connected graph

Note: Figure 9 presents the directed graph extracted from the directionality matrix $\bar{D}_{XY}$ computed in the static case (i.e., for the entire $2000 - 2014$ sample period). The graph's links are oriented according to $\bar{D}_{XY}$, which measures the strength of the information transfer: for a couple of nodes (X,Y), if the element of the matrix $T_{X \to Y}$ is greater than $T_{Y \to X}$, then the edge is oriented from X to Y, otherwise from Y to X. The color of the links indicates intensity with colors ranging from black (directionality index between 0.3 and 0.4), to green (0.4 to 0.5), to blue (0.5 to 0.6) to red (0.6 to 0.7).

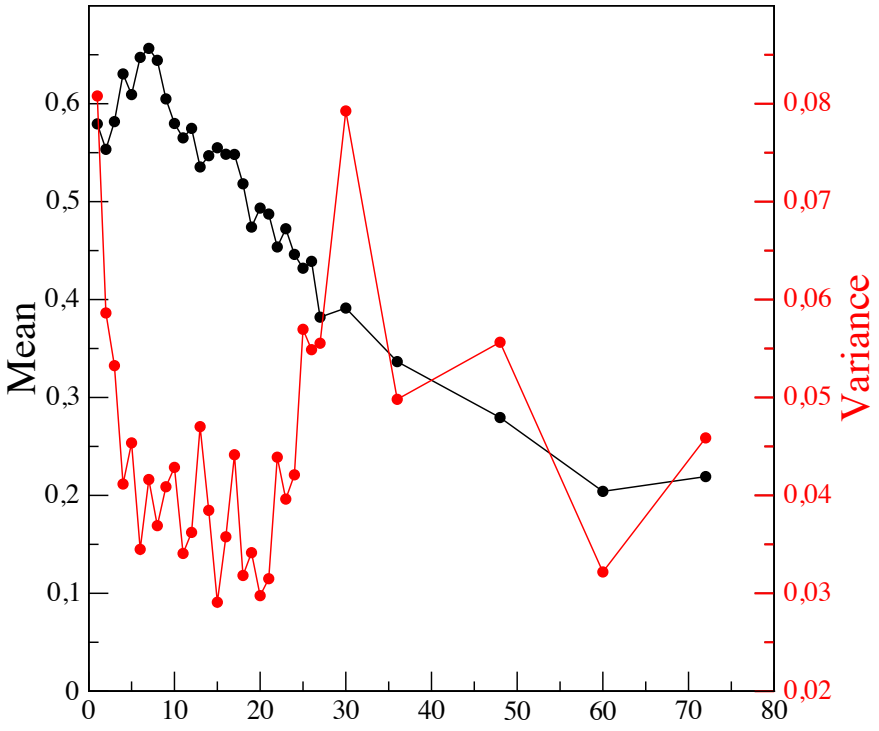

Figure 10: Mean and variance of outflows, static analysis

Note: Figure 10 summarizes our connectivity analysis. Whereas Figure 9 identifies, for each WTI futures contract maturity, which other WTI futures are associated with that maturity, Figure 10 simply plots the mean and variance of the percentage of outer links (i.e., the fraction of all contracts in the WTI term structure) associated with each contract maturity. Sample period: February 2000 to February 2014.

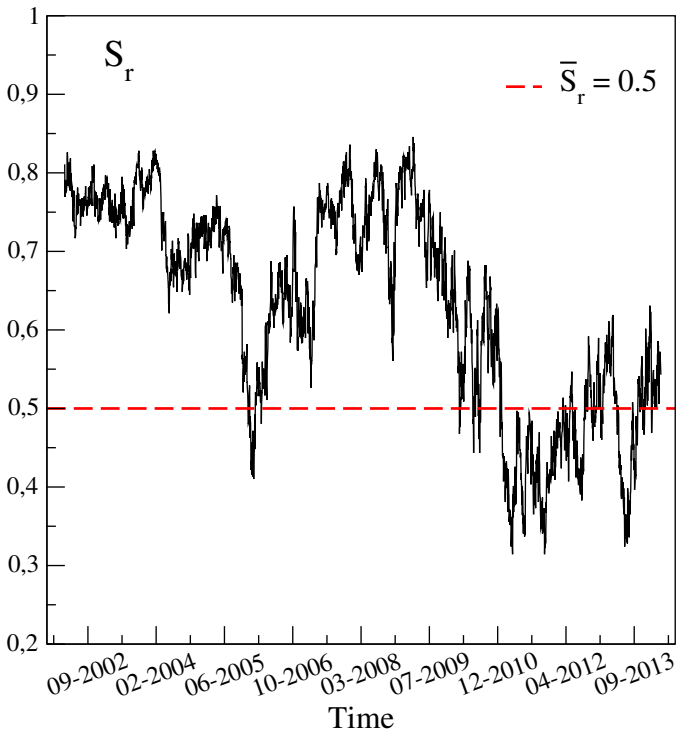

Figure 11: Survival ratios

Note: Figure 11 provides insight into the stability properties of the directed graph in Figure 9 by plotting the survival ratio $\bar{S}_R(t)$. We measure $\bar{S}_R(t)$ as the number of element of same sign in $\frac{1}{N}D_{XY}(t) \cap \bar{D}_{XY}$. On each day t from February 2002 to February 2014, the values are computed using daily returns from the prior two years ($N = 500$ trading days). At one extreme, if $\bar{S}_R(t) = 1$, then the system has the same flow of information as in the static case, i.e., the market is stable. At the other extreme, if $\bar{S}_R(t) = 0$, then the set of directed links has been completely rearranged – indicating disturbances in the flow of information.