Dynamic risk management

Investment, capital structure, and hedging in the presence of financial frictions

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Today's discussion

- Introduction – context, objectives and contribution
- Model description and parameters estimations
- Main results
- Extensions and future work

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Risk management – the fundamentals

- *Absent financial frictions*, risk management does not matter: all "good" projects are financed, investors can diversify idiosyncratic risk, and systematic risk cannot be offloaded below cost (Modigliani and Miller, 1958 and 1963)
- *Asymmetric information* between insiders/managers and outsiders/investors (moral hazard, and/or adverse selection) create financial frictions, hence potential refinancing constraints, hence justify risk management (Holmström and Tirole, 2000)

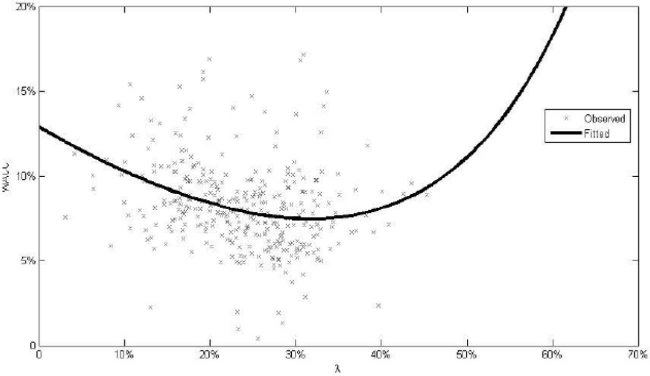
Turning these fundamentals into models of firms' behavior

- *Convex incremental cost of capital*: optimal heding policy function of the correlation between internal wealth and stochastic investment opportunities (Froot, Sharfstein, and Stein, 1993), optimal capital structure (Froot and Stein, 1998)
- Risk management as an *inventory management program* (Rochet and Villeneuve, 2011). "Cash is king": cash reserve is the state variable, bankruptcy occurs when cash runs out. Optimal/maximal cash reserve level, optimal hedging policy: do not hedge past a certain cash reserve
- Add *investment* and *refinancing costs* to the inventory management problem (Bolton, Chen, and Wang, 2011)

Objectives of this work

- Builds on Léautier, Rochet, and Villeneuve, 2007
- Determines a firm's optimal risk management policy (dividend payments, investment level, and "hedging" policy)
  - inventory management program
  - convex cost of capital
  - stochastic investment opportunities

Convex cost of capital



- Refinancing constraint progressively and continously tighter as leverage increases
- Marginal cost of capital applied to the entire capital base, not simply the incremental investment

Stochastic investment opportunities

- A portion of investment is planned (e.g., replenish depreciation)
- Other investments depend on market conditions and opportunities, success of previous ventures, hence are inherently stochastic
- The size of firms is limited by the stochastic creation of opportunities, as well as searching and matching, not only by adjustment costs

Other hypotheses

- Managers maximize the value of the firm, not only shareholders value: consistent with observed practice
- The firm can fully hedge its profit volatility: an oil firm can sell all its production forward, an airline can either purchase its entire oil supply forward or index ticket prices to oil prices
- The firm refinances itself through borrowing only: equity issuance and asset sales are not considered
- The interest rate hence the cost of capital tend to infinity as leverage tends to one
- First two modeling assumptions relaxed and impact of third and fourth discussed in extensions

Main results

- Full hedging is optimal, except for very high leverage: convex capital cost yields concave value function. Full hedging reduces volatility of leverage, hence is optimal ... until gambling for resurrection becomes optimal for very high leverage
- Two target leverage ratios, also fixed points of the leverage dynamics. First, to the left of the cost-minimizing leverage: precautionary savings. Second, on the right of the cost-minimizing leverage: maximum profitable growth
- Results appear robust to relaxing of hypotheses

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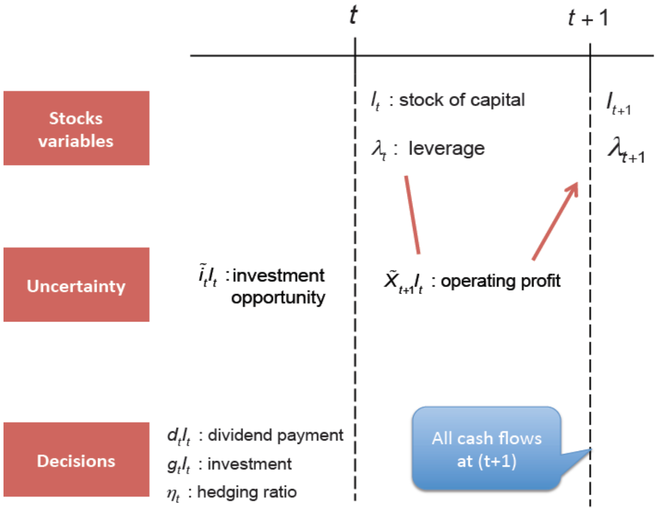
Timing and decisions



Profits and hedging

- Net Operating Profit less Adjusted Taxes (NOPAT):

$$\pi_{t+1} = \tilde{x}_{t+1} I_t$$

- Underlying source of profit uncertainty $\tilde{z}_{t+1}$, i.i.d. and normally distributed
- Costless hedging, forward price equal to the expected spot price:

$$\tilde{x}_{t+1} = \eta_t \mathbb{E}[z] + (1 - \eta_t)\tilde{z}_{t+1},$$

where the hedging ratio η_t satisfies:

$$0 \leq \eta_t \leq 1 \tag{1}$$

Investment and dividends payout

- Investment opportunity $i_t \in \{0, i\}$, i.i.d. Bernouilli trials with probability of arrival p
- Evolution of invested capital

$$I_{t+1} = I_t + g_t I_t - \delta I_t = (1 + g_t - \delta)$$

where $\delta > 0$ is the depreciation rate, and

$$0 \leq g_t \leq \delta + i_t \tag{2}$$

- Resulting Free Cash Flow

$$FCF_{t+1} = (\tilde{x}_{t+1} - g_t + \delta) I_t$$

Evolution of leverage

- Financing flow

$$FF_{t+1} = r(\lambda_t) D_t - (D_{t+1} - D_t) + d_t I_t$$

where D_t book value of debt, $\lambda_t = \frac{D_t}{I_t}$ leverage, $r(\lambda_t)$ interest rate

- Free Cash Flow = Financing Flow

$$(\tilde{x}_{t+1} - g_t + \delta) I_t = r(\lambda_t) D_t - (D_{t+1} - D_t) + d_t I_t$$

- Resulting motion equation

$$\Lambda_{t+1} = 1 - \frac{1 + \tilde{x}_{t+1} - \mu(\lambda_t) - d_t}{1 + g_t - \delta}$$

where $\mu(\lambda_t) = \lambda_t (1 + r(\lambda_t))$

$$\lambda_{t+1} = \begin{cases} 0 & if \quad \Lambda_{t+1} < 0 \\ \Lambda_{t+1} & if \quad 0 \leq \Lambda_{t+1} \leq 1 \\ 1 & if \quad 1 < \Lambda_{t+1} \end{cases} \tag{3}$$

Value of the firm

$$V_t = \mathbb{E}_t \left\{ \sum_{s=t+1}^{\infty} \frac{(x_s - g_{s-1} + \delta) I_{s-1}}{\prod_{k=t}^{s-1} (1 + w(\lambda_k))} \right\}$$

$\iff$

$$v_t = \mathbb{E}_t \left\{ \sum_{s=t+1}^{\infty} \frac{x_s - g_{s-1} + \delta}{\prod_{k=t}^{s-1} (1 + w(\lambda_k))} \prod_{u=t}^{s-2} (1 + g_u - \delta) \right\}$$

where $v_t = \frac{V_t}{I_t}$ is the relative value of the firm (average Tobin's Q)

Bellman equations

- $J_t(\lambda_t, i_t)$ the value of the firm for the optimal controls is recursively defined by:

$$J_t(\lambda_t, i_t) = \max_{\substack{g_t,\eta_t,d_t \\ s.t.\ (1),\ (2),\ (3)}} \frac{1}{1+w(\lambda_t)} \left\{ \begin{array}{c} \mathbb{E}[z] - g_t + \delta + \\ (1+g_t-\delta)\mathbb{E}_t\left\{J_{t+1}\right\} \end{array} \right\}$$

- Terminal value:

$$\begin{aligned} J_{T+1}\left(\lambda_{T+1}, i_{T+1}\right) &= \frac{\mathbb{E}[z] - g + \delta}{1+w\left(\lambda_{T+1}\right)} \sum_{s=T+2}^{\infty} \left(\frac{1+g-\delta}{1+w(\lambda_{T+1})}\right)^{s-2-T} \\ &= \frac{\mathbb{E}[z] - (g-\delta)}{w(\lambda_{T+1}) - (g-\delta)} \end{aligned}$$

where $(g-\delta)$ is the long-term net growth rate

First-order derivatives

- Define

$$\Theta_t = \frac{1}{1+w(\lambda_t)} \left\{ \mathbb{E}[z] - g_t + \delta + (1 + g_t - \delta)\mathbb{E}_t \left\{ J_{t+1} \right\} \right\}$$

- Then:

$$\frac{\partial \Theta_t}{\partial d_t} = \frac{1}{1+w(\lambda_t)} \mathbb{E}_t \left\{ \frac{\partial J_{t+1}}{\partial \lambda_{t+1}} \right\}$$

$$\frac{\partial \Theta_t}{\partial g_t} = \frac{1}{1+w(\lambda_t)} \left[-1 + \mathbb{E}_t \left\{ J_{t+1} \right\} + \mathbb{E}_t \left\{ \frac{\partial J_{t+1}}{\partial \lambda_{t+1}} \left(1 - \tilde{\lambda}_{t+1}\right) \right\} \right]$$

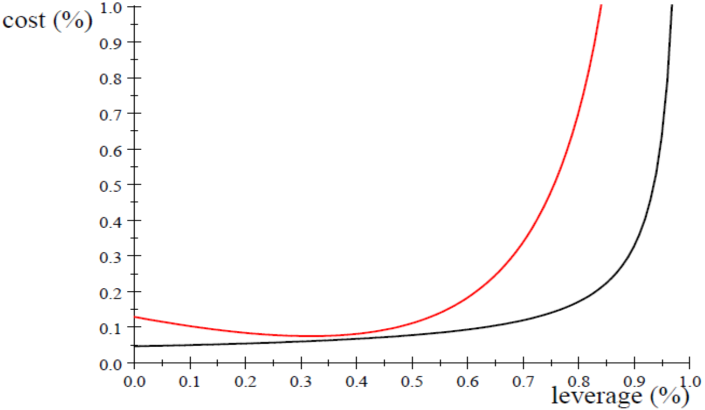
$$\frac{\partial \Theta_t}{\partial \eta_t} = \frac{1}{1+w(\lambda_t)} \mathbb{E}_t \left\{ \frac{\partial J_{t+1}}{\partial \lambda_{t+1}} \left(\tilde{z}_{t+1} - \mathbb{E}\left[z\right] \right) \right\}$$

Estimation of the parameters

Statistical analysis of annual data for a 20 year panel of 854 industrial firms

$g - \delta$	long-term net growth rate	2.1%
δ	depreciation	12%
i	investment opportunity	14.7%
p	prob. of arrival of investment opp.	21.2%
$\mathbb{E}[z]$	expected *ROIC*	8.4%
σ_z	std. dev. of *ROIC*	5.8%

Estimated cost of capital and (after tax) interest rate



$\bar{\lambda} \approx 31.7\%$ minimizes the cost of capital

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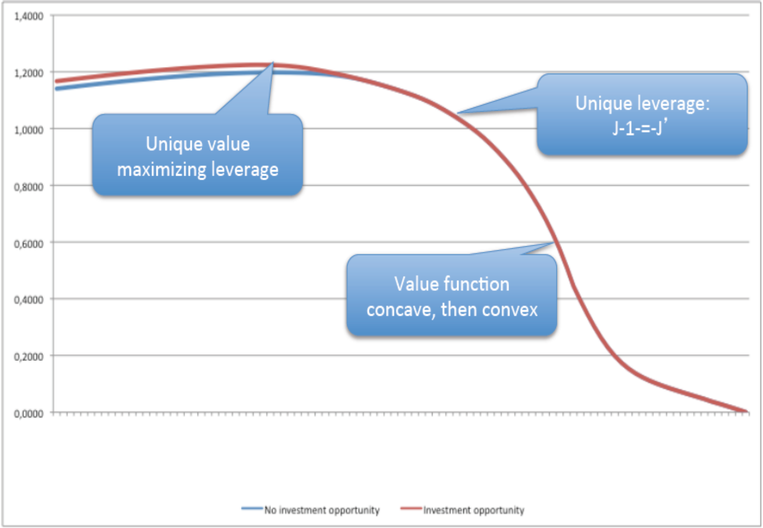
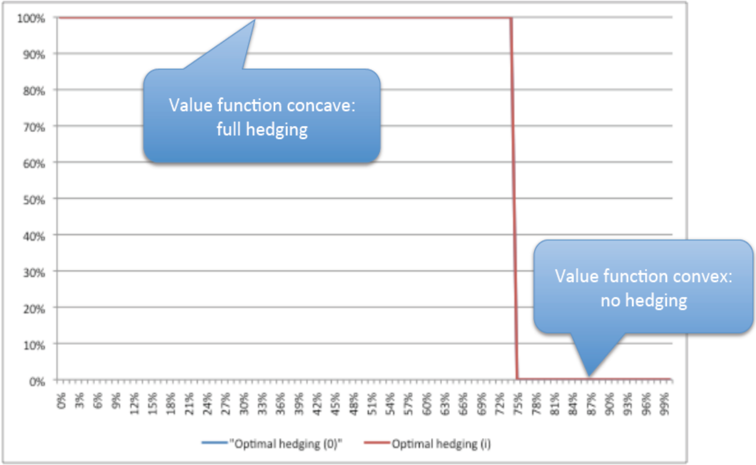
(Stationary) value function

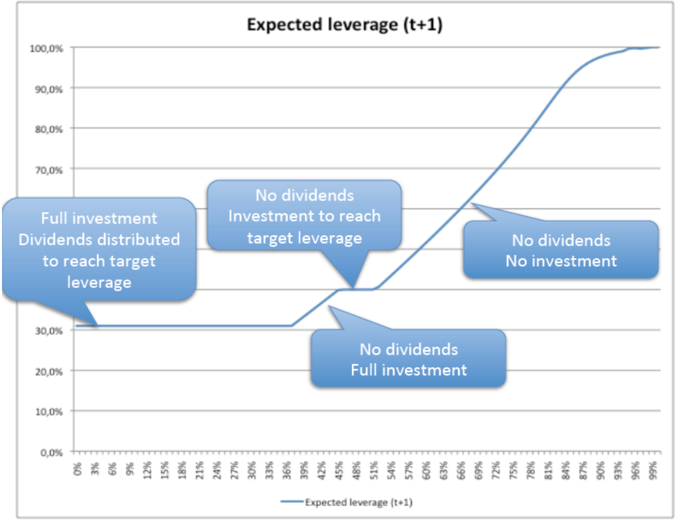


Optimal hedging policy

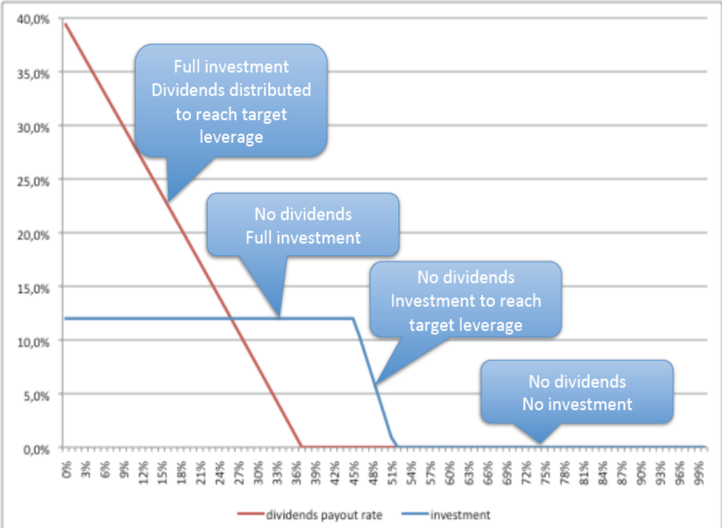


For $\lambda_{t+1} \leq b_{t+1}$, J_{t+1} is concave, hence $\eta_t = 1$: λ_{t+1} is deterministic

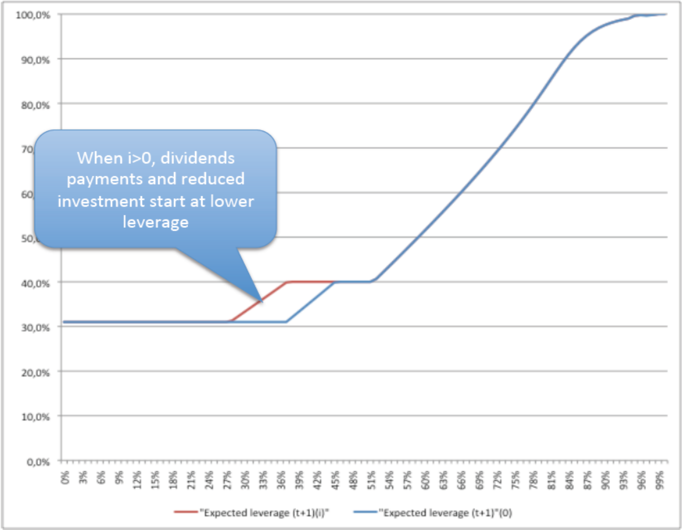
Next period (expected) leverage



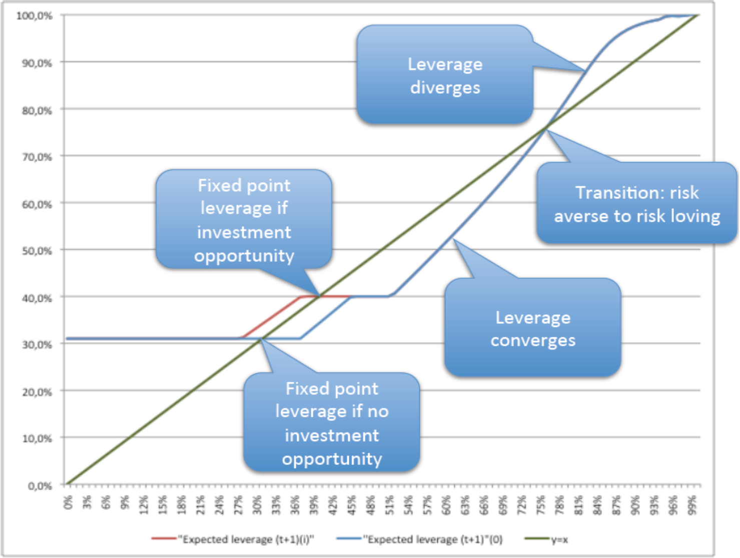
Optimal dividend and investment policies



Impact of investment opportunity in current period



Leverage dynamics



Sketch of the proof

- Backwards induction from $(T+1)$
- As long as there is no truncation at the optimum:

$$\begin{aligned} \lambda_{t+1} &= \frac{\mu(\lambda_t) + g_t - \delta + d_t - \mathbb{E}[z]}{1 + g_t - \delta} + \frac{(1-\eta_t)(\tilde{z}_{t+1} - \mathbb{E}[z])}{1 + g_t - \delta} \\ &= y_{t+1} + (1-\eta_t)\,\tilde{\varepsilon}_{t+1} \end{aligned}$$

- Define

$$\varphi_{t+1}(y_{t+1}) = \mathbb{E}_t\left\{J_{t+1}(\lambda_{t+1}, \tilde{\imath}_{t+1})\right\}$$

- If $\mathbb{E}[z] - w(\bar{\lambda}) > 0$, $\varphi_{T+1}(y_{T+1})$ satisfies

$$\begin{aligned} &\varphi_{T+1}(y_{T+1}) \text{ concave, unique maximum } \bar{\lambda}_{T+1} \in [a_{T+1}, b_{T+1}] \subset [0,1] \\ &\exists! \hat{\lambda}_{T+1} \in \left[\bar{\lambda}_{T+1}, b_{T+1}\right] \,/\, \left\{ \begin{array}{c} \varphi_{T+1}(x) - 1 + (1-x)\,\varphi'_{T+1}(x) \geq 0 \\ \Leftrightarrow x \geq \hat{\lambda}_{T+1} \end{array} \right\} \end{aligned} \qquad (\mathcal{P})$$

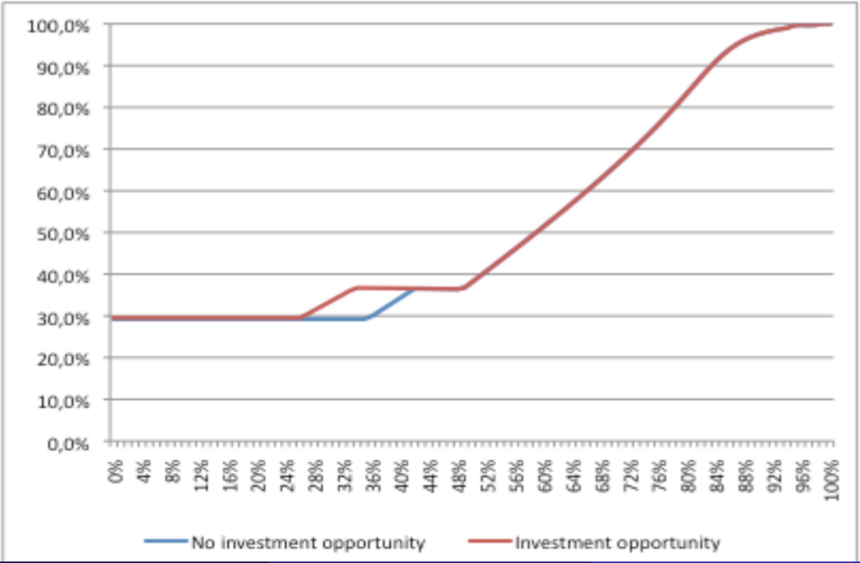
- Suppose $\varphi_{t+1}(y_{t+1})$ satisfies property $(\mathcal{P})$. Then, we (1) determine the optimal controls at date t, and (2) prove that $\varphi_t(y_t)$ satisfies property $(\mathcal{P})$

Larger investment opportunity

- $i = 20\%$ instead of 14.6%
- Same solution structure
- Higher precautionary savings: low target leverage 30% instead of 31%
- Higher value: maximum value 3% higher

No hedging restriction: same solution structure

Expected next period leverage



Impact of hedging restriction

- Higher precautionary savings: low target leverage 29.3% instead of 31%. "Hedging is tax advantaged equity"
- Lower value: maximum value 8% lower when hedging restricted. Consistent with some empirical estimates (Allayanis and Weston, 2001)

- Suggests structure of solution unchanged if we add a non-hedgeable risk

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Maximizing shareholder value

- Shareholder value $\mathcal{S}_t = \frac{1}{1+w(\lambda_t)} \left(d_t I_t + \mathcal{S}_{t+1} \right)$
- Bellman equations for shareholder value per unit of equity $S_t = \frac{\mathcal{S}_t}{(1-\lambda_t) I_t}$

$$(1-\lambda_t) S_t = \frac{1}{1+w(\lambda_t)} \max_{d_t, g_t, \eta_t} \left[\begin{array}{c} d_t + \\ (1+g_t-\delta) \mathbb{E}_t \left\{ (1-\lambda_{t+1}) S_{t+1} \right\} \end{array} \right]$$

- Terminal value

$$S_{T+1}(\lambda_{T+1}) = \frac{1}{(1-\lambda_{T+1})} \left(J_{T+1}(\lambda_{T+1}) - \lambda_{T+1} \frac{1+r(0)}{1+r(\lambda_{T+1})} \right)$$

- Preliminary analysis suggest the same solution structure

Other next steps

- Equity issuance: fixed and variable costs, relevant only for high leverage
- Asset sales: (1) reserve price (decreasing with leverage), (3) bargaining between buyer and seller, but also (2) stochastic sale opportunity
- Correlation between profits and investment opportunities, serially correlated profits, non-normal distributions
- "Large risks": no truncation assumptions violated