Supply and Shorting in Speculative Markets

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Motivation for this paper

I

1. Models with heterogeneous beliefs and no-short-sales (Harrison and Kreps (1978)) are capable of generating the observed correlation between bubbles and trading volume (Scheinkman and Xiong (2003) or Berestycki et al. (2016))
2. Bubble implosions often coincide with increases in supply of assets.
    - Two period model with risk-aversion (Hong et al. (2006))

Motivation for this paper II

3. Changes in costs of shorting and the implosion of bubble in MBS. (Lewis (2015))
   - CDO machine: transform Home Equity Bonds based in less than prime mortgages, mostly with ratings AA, A or BBB, into AAA rated bonds.
   - Tools: tranching, correlation assumptions, using mezzanine CDO tranches as collateral for other CDOs.
   - Cordell et al. (2011) estimates that virtually all CDO tranches *sold to investors* had AAA ratings ($CDO^2, \ldots$).
   - Innovations: standardized CDS on MBS's (summer 2005), standard CDS on CDO tranches (summer 2006).

Motivation for this paper III

- Synthetic CDOs using CDSs as collateral, mostly issued since 2005H2, more than doubled the amount of BBB tranches of Home Equity Bonds placed in CDOs during 1998-2007 (Cordell et al. (2011), Table 3.)
- Abacus 2007-AC1, the synthetic CDO made infamous by the SEC enforcement action against Goldman Sachs, was composed of CDS totaling $2 billion. The original cash value of the underlying BBB bonds was $1.238 billion.
- Model in paper allows for shorting and decrease in cost of shorting decreases prices and may turn positive bubble into negative bubble.

Difference in beliefs and asset pricing

I

- Risk neutral agents
- Agents can borrow and lend at a constant (risk-less) rate.
- Some state variable X that determines future payoffs
- Agents agree to disagree on the future evolution of X.
  - In S-X agents overconfident about value of distinct signals
- No shorting
- Buyer acquires right to dividends and right to sell the asset to agents that value the asset more. (*Resale option*).
- Buyer pays more for asset than she would pay if restricted to buying at time-zero and holding to maturity.

Difference in beliefs and asset pricing II

- Since agents are risk-neutral, buyer is indifferent regarding amount purchased. Supply of asset is irrelevant.
- In dynamic set-up buyer has opportunity to delay purchases. This should depress prices relative to when agents are restricted to buy and hold strategy. However since buyers are risk-neutral, in equilibrium, they attribute zero value to this *Delay option.*
- Consequence: Resale option and absence of value of delay option guarantee equilibrium price is larger than equilibrium price when agents cannot re-trade (Bubble).

Description of model and main results

- Add (quadratic) cost-of-carry.
  - $$c(y) = \frac{1}{2\alpha_+} y^2, \; y \geq 0$$
  - Quadratic costs reflect not only monetary costs but also risks that liquidity shocks force agents to liquidate.
  - Marginal valuation of asset is a decreasing function of asset holdings.
  - Cheap risk-aversion
  - Supply matters
    - Search frictions Duffie et al. (2005)
- Supply exogenous

Description of model and main results II

- Allow for shorting
- $$c(y) = \frac{1}{2\alpha_-}y^2,\ y \leq 0$$
    - Larger trading costs, $\alpha_- \leq \alpha_+$.
    - D'Avolio (2002)
    - Earlier literature corresponds to $\alpha_- = 0$ and $\alpha_+ = \infty$
    - Complete markets model of Dumas et al. (2009).

Description of model and main results III

- Results:
  1. Characterize equilibria as solution of PDE
     - Existence and uniqueness
     - Comparison of solutions
  2. Equilibria solve optimization problem faced by a planner with limited instruments.
  3. If shorting is costly ($\alpha_- < \infty$), resale option has value.
  4. If going long has zero carrying cost ($\alpha_+ = \infty$) and $\alpha_- < \infty$ price is independent of cost of going short.
     - Delay option has no value.
     - Crucial assumption in early literature is not "no-shorting" but "shorting costly and marg. cost of long constant."

Description of model and main results

IV

5. Comparison of equilibrium price to price when agents can only buy at time zero and hold to maturity.
   - Difference identified as bubble (value of speculation).
   - Example shows that even when shorting not allowed, if $\alpha_+ < \infty$, delay option may dominate resale option and result in negative bubble.
   - New in this literature, but similar results observed in papers on search frictions. (Lagos and Rocheteau (2006), Feldhütter (2012) and Hugonnier et al. (2014))
   - Examples also show that lowering the cost of shorting may cause a positive bubble to become negative.
6. Effect of adding linear terms to costs.

Model

- $n \geq 1$ types, each with a unit measure of agents, who trade a security over a finite time interval $[0, T]$.
- Single payoff $f(X(T))$, $f : \mathbb{R}^d \to \mathbb{R}$ bounded continuous.
- Agents agree to disagree on evolution of X.
- X to be the coordinate-mapping process on the space $\Omega = C([0, T], \mathbb{R}^d)$ of continuous, d-dimensional paths, equipped with the canonical filtration and σ-field.

Model II

- Type i has probability measure Q_i on Ω under which

$$dX(t) = b_i(t, X(t))\,dt + \sigma_i(t, X(t))\,dW_i(t), \quad X(0) = x, \tag{1}$$

where W_i is a Brownian motion of dimension d' and

$$b_i : [0, T] \times \mathbb{R}^d \to \mathbb{R}^d, \quad \sigma_i : [0, T] \times \mathbb{R}^d \to \mathbb{R}^{d \times d'}$$

are deterministic. For simplicity, b_i and σ_i are in $C_b^{1,2}$.

Model

III

- Allow differences in drift and volatility. Plenty of evidence that complex processes involving stochastic and time-varying volatility are necessary to understand empirical features of asset prices. In this context, agents may also differ in their forecasts of volatility.
- Write A^2 for the product $AA^\top$ of A with its transpose $A^\top$. σ_i^2 uniformly parabolic; that is eigenvalues $>> 0$ are uniformly bounded away from zero.
- These conditions imply (1) has a unique (strong) solution.
- Agents risk-neutral and have access to a risk-free asset whose interest rate is normalized to zero.

Model IV

- Trade the security throughout the interval $[0, T]$, at a time t price $P(t)$ determined in equilibrium.
- *Admissible* portfolio Φ for an agent is a bounded progressively measurable process. Write $\Phi \in \mathcal{A}$.
- $\Phi(t)$ number of units of the security held by the agent at time t, (< 0 for a short position.)

Model

V

- Agents subject to instantaneous cost-of-carry c

$$c(y) = \begin{cases} \frac{1}{2\alpha_+} y^2, & y \geq 0, \\ \frac{1}{2\alpha_-} y^2, & y < 0. \end{cases} \tag{2}$$

Here the (inverse) cost coefficients $\alpha_\pm$ given constants satisfying $0 < \alpha_- \leq \alpha_+$.
Given a (semimartingale) price process P, agent i seeks to maximize the expected net payoff

$$E_i \left[\int_0^T \Phi(t)\, dP(t) - \int_0^T c(\Phi(t))\, dt \right]; \tag{3}$$

Model

VI

- $\Phi_i \in \mathcal{A}$ is *optimal* for type i if it maximizes (3) over all $\Phi \in \mathcal{A}$. Study symmetric equilibria in which every agent of a type chooses same portfolio.
- Supply owned by third parties that supply their endowment inelastically. $S(t) = s(t, X(t)) \geq 0$ $s \in C_b^{1,2}$ (*supply function*)

Equilibrium

- P continuous, progressively measurable, semimartingale, with $P(T) = f(X(T))$ (Q_i a.s.) is an *equilibrium price* if there are $\Phi_i \in \mathcal{A}$, Φ_i optimal for type i and

$$\sum_{i=1}^{n} \Phi_i(t) = S(t)$$

- Markovian equilibria: $P(t) = v(t, X(t))$ for a deterministic function v (*equilibrium price function.*)
- Expected instantaneous change in v (for type i) $\mathcal{L}^i v(t,x) = \partial_t v(t,x) + b_i \partial_x v(t,x) + \frac{1}{2} \operatorname{Tr} \sigma_i^2 \partial_{xx} v(t,x)$.

Equilibrium

II

- Homogeneous beliefs: $b_i = b_j$ and $\sigma_i = \sigma_j$ for all (i, j).
- In equilibrium all agents are long, $\mathcal{L}^i v(t,x) \geq 0$ and quadratic cost implies $\frac{\Phi_i(t)}{\alpha_+} = \mathcal{L}^i v(t, X(t))$ for each i.
- $\Phi_i(t) = \phi_i(t, X(t))$ with $\phi_i(t,x) = \alpha_+ \mathcal{L}^i v(t,x)$.
- Since market clearing requires $n\phi_i(t,x) = s(t,x)$,

$$\begin{aligned} \partial_t v(t,x) + b_i \partial_x v(t,x) + \frac{1}{2} \operatorname{Tr} \sigma_i^2 \partial_{xx} v(t,x) - \frac{s(t,x)}{n\alpha_+} &= 0 \\ v(T,x) &= f(x) \end{aligned}$$

- Supply as a running cost: equilibrium price must rise to compensate for carrying cost.

Equilibrium III

- $I \subset \{1, \ldots, n\}$, I^c, $|I|$. Interpretation $i \in I$ goes short.
- Average coefficients associated with I

$$b_I = \frac{1}{|I|} \sum_{i \in I} b_i, \quad b_{I^c} = \frac{1}{|I^c|} \sum_{i \in I^c} b_i,$$

$$\mu_I = \frac{|I|\alpha_-}{|I|\alpha_- + |I^c|\alpha_+} b_I + \frac{|I^c|\alpha_+}{|I|\alpha_- + |I^c|\alpha_+} b_{I^c},$$

$$\sigma_I^2 = \frac{1}{|I|} \sum_{i \in I} \sigma_i^2, \quad \bar{\sigma}_{I^c}^2 = \frac{1}{|I^c|} \sum_{i \in I^c} \sigma_i^2,$$

$$\Sigma_I^2 = \frac{|I|\alpha_-}{|I|\alpha_- + |I^c|\alpha_+} \sigma_I^2 + \frac{|I^c|\alpha_+}{|I|\alpha_- + |I^c|\alpha_+} \sigma_{I^c}^2,$$

$$\kappa_I(t, x) = \frac{s(t, x)}{|I|\alpha_- + |I^c|\alpha_+}.$$

Theorem

(i) There exists a unique equilibrium price function $v \in C_b^{1,2}$. The optimal portfolios are unique $\Phi_i(t) = \phi_i(t, X(t)) = \alpha_{\mathrm{sign}(\mathcal{L}^i v(t,x))} \mathcal{L}^i v(t,x)$.
(ii) The function $v \in C_b^{1,2}$ is the unique solution of

$$\partial_t v(t,x) + \sup_I \Big(\mu_I(t,x) \partial_x v(t,x) + \tfrac{1}{2} \operatorname{Tr} \Sigma_I^2(t,x) \partial_{xx} v(t,x) - \kappa_I(t,x) \Big) = 0 \tag{4}$$

on $[0,T) \times \mathbb{R}^d$ with $v(T,x) = f(x)$

- The equilibrium price $v(t,x)$ is 0-homogeneous in (α_-, α_+, s).
- "Running cost" κ_I depends only on $|I|$. $\alpha_- \leq \alpha_+ \rightarrow$ running costs $\nearrow$ with $|I|$.

Optimal control formulation

- Θ be the collection of all progressively measurable processes $\mathcal{I}$ with values in family of subsets of $\{1, \ldots, n\}$.
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$$\begin{aligned} \sup_{\mathcal{I}\in\Theta} \quad & E \quad \left[f(X^{t,x}_{\mathcal{I}}(T)) - \int_0^T \kappa_{\mathcal{I}(r)}(r, X^{t,x}_{\mathcal{I}}(r))\, dr \right] \qquad (5) \\ \text{where } X(t) &= x \text{ and} \\ dX^{t,x}_{\mathcal{I}} &= \mu_{\mathcal{I}(r)}(r, X(r))\, dr + \Sigma_{\mathcal{I}(r)}(r, X(r))\, dW(r) \end{aligned}$$

Optimal control formulation II

- *Theorem: The equilibrium price function v coincides with the value function V of (5). Moreover, an optimal control for (5) is given by $\mathcal{I}_*(t) = I_*(t, X(t))$, where*

$$I_*(t,x) = \{i \in \{1,\dots,n\} : \mathcal{L}^i v(t,x) < 0\}. \tag{6}$$

Planner with limited instruments

- Planner assigns to each type at each date event pair, a "total cost coefficient" $\alpha_i \in \{\alpha_-, \alpha_+\}$. If an agent of a type is assigned α_i, her effective cost is $\frac{1}{2\alpha_i} y^2$ independent of the sign of y.
- Planner subsidizes (taxes) agents.
- Prices clear markets.
- Aggregate taxes collected by the planner are returned lump-sum to agents.
- Planner's objective is to maximize the initial price.

Planner with limited instruments II

- Since taxes are returned lump-sum, agents to maximize the expected (net of lump-sum transfers) payoff

$$E_i\left[\int_0^T \Phi(t)\,dP(t) - \int_0^T c_i(t, X(t), \Phi(t))\,dt\right]$$

- Cost-of-carry is given by the assigned coefficient,

$$c_i(t,x,y) = \frac{1}{2\alpha_i(t,x)}y^2,$$

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$$I(t,x) = \{i \in \{1,\dots,n\} : \alpha_i(t,x) = \alpha_-\}.$$

Planner with limited instruments III

- Theorem: *(i) For any assignment $\mathcal{I}(t) = I(t, X(t))$ of the planner, there exists a unique equilibrium price process $P_{\mathcal{I}}(t) = v_{\mathcal{I}}(t, X(t))$ with $v_{\mathcal{I}} \in C_b^{1,2}$, and $v_{\mathcal{I}}$ is given by*

$$v_{\mathcal{I}}(t,x) = E\left[f(X_{\mathcal{I}}^{t,x}(T)) - \int_0^T \kappa_{\mathcal{I}(r)}(r, X_{\mathcal{I}}^{t,x}(r))\, dr \right]. \tag{7}$$

(ii) If the planner's aim is to maximize the price $P_{\mathcal{I}}(0) = v_{\mathcal{I}}(0,x)$, then the assignment (6) is optimal and $V(0,x)$ of (5) is the optimal price.

Planner with limited instruments

IV

- In the optimal assignment, *no taxes or subsidies are collected* - in the resulting equilibrium, agents assigned α_- go short and agents assigned α_+ go long.
- Planner penalizes pessimists by assigning the higher cost-of-carry to short positions, which then leads to the maximal price that the planner can produce given her constraints.

Comparison of solutions and limiting cases

I

- *The equilibrium price function v is monotone decreasing with respect to the supply function s.*
- *The equilibrium price function v is*
  1. *increasing with respect to* α_+
  2. *decreasing with respect to* α_-
  3. *increasing with respect to the quotient* α_+/α_- *if* $s \equiv 0$.
- Proofs use comparison principle.

Comparison of solutions and limiting cases II

- Previous literature assumes $\alpha_+ = \infty$ and $\alpha_- = 0$.
- *As* $\alpha_+ \to \infty$, v^{α_-,α_+} *converges to unique solution of*

$$\partial_t v + \sup_{i \in \{1,\ldots,n\}} \Big(b_i \partial_x v + \tfrac{1}{2} \operatorname{Tr} \sigma_i^2 \partial_{xx} v \Big) = 0$$

with $v(T,x) = f(x)$. *The convergence is locally uniform in* (t,x) *and monotone increasing.*

- Limit v^∞ is independent of α_- and s.
- Proof relies on stability of value functions (*e.g.* Krylov, *Controlled Diffusion Processes*)

Comparison of solutions and limiting cases III

- *As* $\alpha_- \to 0$, v^{α_-,α_+} *converges to unique solution of*

$$\partial_t v + \sup_{\emptyset \neq J \subseteq \{1,\dots,n\}} \left(\frac{1}{|J|} \sum_{i\in J} b_i \partial_x v + \frac{1}{2} \operatorname{Tr} \frac{1}{|J|} \sum_{i \in J} \sigma_i^2 \partial_{xx} v - \frac{s}{|J|\alpha_+} \right) = 0$$

with $v(T,x) = f(x)$. *(Also l.u. and monotone)*

Static equilibrium

- Buy and hold strategies at $t = 0$.
- $s =$ constant
- $$p_{sta} = \max_{I \subseteq \{1,\dots,n\}} \left(\frac{\alpha_-}{|I|\alpha_- + |I^c|\alpha_+} \sum_{i \in I} e_i + \frac{\alpha_+}{|I|\alpha_- + |I^c|\alpha_+} \sum_{i \in I^c} e_i - \frac{sT}{|I|\alpha_- + |I^c|\alpha_+} \right)$$

  where $e_i = E_i[f(X(T))]$.
- As $\alpha_+ \to \infty$ static prices converge to

$$p_{sta}^{\infty} = \max_{i \in \{1,\dots,n\}} E_i[f(X(T))]$$

  - Most optimist determines price if it is costless to go long.

Effect of speculation

- Difference between $v(0,\cdot)$ and p_{sta} (bubble)
- Two options for long party
  1. Option to resell. Makes $v(0,\cdot) > p_{\mathrm{sta}}$ (To have value requires $\alpha_- < \infty$.)
  2. Option to delay. Makes $v(0,\cdot) < p_{\mathrm{sta}}$ (To have non-zero impact requires $\alpha_+ < \infty$.)
     - Since agent are risk-neutral if there is no cost of going long buyer must be indifferent as to how much she wants to buy.
- Corresponding options for short party, with reverse implications.

Effect of speculation

II

- Example shows that even when shorting is not allowed if $\alpha_+ < \infty$, the delay option may dominate the resale option and result in a negative bubble.
- Examples also show that lowering the cost of shorting may cause a positive bubble to become negative.
    - MBS indexes and synthetic CDO's.

Adding linear costs

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$$c(y) = \begin{cases} \frac{1}{2\alpha_+} y^2 + \beta_+ y, & y \geq 0, \\ \frac{1}{2\alpha_-} y^2 + \beta_- |y|, & y < 0, \end{cases} \tag{8}$$

- Groups I (shorts) and J (strictly longs) with J perhaps a proper subset of I^c
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$$\partial_t v(t,x) + \sup_{I \cap J = \emptyset} \Big(\mu_{I,J}(t,x) \partial_x v(t,x) + \tfrac{1}{2} \operatorname{Tr} \Sigma^2_{I,J}(t,x) \partial_{xx} v(t,x) - \kappa_{I,J}(t,x) \Big) = 0$$

- Comparison of solutions and limit theorems go through.

Sketch of proofs

- Given function $v \in C_b^{1,2}$ set $I_*(t,x) = \{i \in \{1,\dots,n\} : \mathcal{L}^i v(t,x) < 0\}$.
- Given $P(t) = v(t,x)$ $v \in C_b^{1,2}$ agent of type i chooses $\phi_i(t,x) = \alpha_{\text{sign}(\mathcal{L}^i v(t,x))} \mathcal{L}^i v(t,x)$. (Ito's lemma)
- Market clearing condition: $\alpha_- \sum_{i \in I_*} \mathcal{L}^i v + \alpha_+ \sum_{i \in I_*^c} \mathcal{L}^i v = s$.
- $\alpha_- \leq \alpha_+$. If $i \in I_*$, then $\mathcal{L}^i v \leq 0 \Rightarrow \alpha_- \mathcal{L}^i v \geq \alpha_+ \mathcal{L}^i v$. If $i \in I_*^c$, then $\mathcal{L}^i v \geq 0$ and $\alpha_+ \mathcal{L}^i v \geq \alpha_- \mathcal{L}^i v$.
- $\max_{I \subseteq \{1,\dots,n\}} \left(\alpha_- \sum_{i \in I} \mathcal{L}^i v + \alpha_+ \sum_{i \in I^c} \mathcal{L}^i v - s\right) = 0$ and the set I_* is a maximizer.

Sketch of proofs II

- $\max_{I \subseteq \{1,\ldots,n\}} \frac{1}{|I|\alpha_- + |I^c|\alpha_+} \left(\alpha_- \sum_{i \in I} \mathcal{L}^i v + \alpha_+ \sum_{i \in I^c} \mathcal{L}^i v - s \right) = 0.$
- Plug in the definition of $\mathcal{L}^i v$ and use the definitions of μ_I, Σ_I and κ_I, to obtain the PDE (4).
- Converse is straightforward
- Comparison of solutions
    - $\alpha'_+ > \alpha_+ \Rightarrow \alpha_- \sum_{i \in I_*} \mathcal{L}^i v + \alpha'_+ \sum_{i \in I_*^c} \mathcal{L}^i v - s \geq 0.$
    - $\max_{I \subseteq \{1,\ldots,n\}} \left(\alpha_- \sum_{i \in I} \mathcal{L}^i v + \alpha'_+ \sum_{i \in I^c} \mathcal{L}^i v - s \right) \geq 0$
    - $\max_{I \subseteq \{1,\ldots,n\}} \frac{1}{|I|\alpha_- + |I^c|\alpha_+} \left(\alpha_- \sum_{i \in I} \mathcal{L}^i v + \alpha'_+ \sum_{i \in I^c} \mathcal{L}^i v - s \right) \geq 0.$

Sketch of proofs

III

- Plug in the definition of $\mathcal{L}^i v$ and use the definitions of μ_I', Σ_I' and κ_I', to obtain

$$\partial_t v(t,x) + \sup_I \Big(\mu_I'(t,x)\partial_x v(t,x) + \tfrac{1}{2}\operatorname{Tr}(\Sigma_I')^2(t,x)\partial_{xx} v(t,x) - \kappa_I'(t,x) \Big) \geq 0.$$

  on $[0,T) \times \mathbb{R}^d$ with $v(T,x) = f(x)$.
- Note that the sign convention chosen here is opposite to the one of Fleming and Soner (2006), so that a subsolution corresponds to the inequality ≥ 0 in the PDE. Thus v is a subsolution for the PDE with the new parameter values.

Sketch of proofs IV

- Let v' be the solution for the PDE with the new parameter values. As v and v' satisfy the same terminal condition f, the comparison principle (Fleming and Soner, 2006, Theorem V.9.1, p. 223) implies that $v \leq v'$.

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