Some Mathematical Tools for Modeling Water Management

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Water : An ancient, intricate, and challenging management issue

- Water has always been a scarce resource in many regions
- Managing water supply and usage has always been a critical task :
- A tough complex intricate challenge for stakeholders
  - Temporal, seasonal, and spatial issues
  - Highly diversified supplies, demands, uses.
  - From freshwater to industrial consumption
  - From distant to local supply
  - Allocation mediated sometimes by the market or negotiation,
  - but at times also incites conflicts and wars

Some issues, from a modeling view point

- Most of the time : mix of market and regulation
- Stochastic high dimensional problems

- Governance is often one of the elements of the problem
- As demonstrated by the works of Elinor Ostrom

What's new in decision support tools ?

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- There is now
- a complete suite of quantitative math tools
- For modeling and decision support
- for all, or nearly all, issues related to
- the management and allocation of a common good,
- like water.

The usual framework

- The stakeholders choose the governance and their representatives.
- governance and representatives decide regulation
- Representatives choose the regulations and the marketplaces
- Given regulations and markets, an equlibrium is achieved
- Hence, through the voting system, stakeholders express the opinion that optimizes their preferences within the anticipated equilibrium.
- This close the loop

- Let’s see an example, in the vein of a Robinson Crusoe narrative
- (real life modeling takes much more time)

A stylized example

- An isolated region populated by farmers.
- All residents draw water from the same aquifer.
- Each resident can choose their type K of well, and maintenance cost is $I = cK$.
- The flow of production value of agent a is given by a function $F(a, K, X, \theta, q)$
- Which depends
  - on the level X of the groundwater table,
  - on the characteristics a and K of their personal installation,
  - On the quantity q of water drawn per day,
  - And on the collective regulatory function θ (e.g., θ(X) = the allowed daily watering duration based on the level of the groundwater table).
- Ex. : $F(a, K, X, \theta, q) = \sqrt{KXa\ Min(\theta, q)}$

Water table dynamics : climate hazard and farmers' withdrawals

The water table level depends on farmers' withdrawals and climatic hazards.

Let's assume that the dynamics of the water table are given by :

$$dX = -\int q_t^a dm_0(a)\, dt + \alpha(X - \beta)dt + \sigma dW$$

Where m_0 represents the distribution of agents types, and

Where $\alpha(X-\beta)dt+\sigma dW$ is an O-U type random dynamics.

The equilibrium is defined by the agents decisions, while agents anticipate the collective equilibrium.

- For a specific regulation θ, each agent a optimizes the choice of their facility K, aware that for a chosen K, they will further optimize their withdrawal strategy to maximize :

$$Max\ \mathbb{E}\ P = \int_0^{\infty} e^{-rt} F(a, K, \theta(X_t), q_t) dt$$

Note: to reduce complexity, one can replace P by the ergodic criteria

$P_0 = \mathbb{E} \int F(a, K, \theta(x), q(x)) d\mu(x)$

Where μ is the stationary measure over the set of states of the water table generated by the set of optimal behaviors $Q(a, K, \theta, \mu, x)$ of agents.

Despite complexity and dimensionnality, There are now efficient solvers

- There are now numerical methodologies and solvers
- Based on efficient usage of neural networks
- To compute the previous equilibrium

- i-e : to solve the Master equation of the previous MFG framework

- But of course, implementation of these new quantitative modeling capacities requires tough time-consuming work in each specific case

Beyond the previous example

- Water supply management problems are extremely diversified :
- Several populations, needs, risks, suppliers,..
- Almost any concepts of optimization and of game theory
- Appears in some specific contexts
- Master equations and their solvers are interoperable
- With almost all concepts of game theory

One more thing : the governance challenge

- Elinor Ostrom has shown the importance of governance design
- For management of water supply

- Can we develop quantitative modeling solutions
- to aid in the assessment of governance design?

Let’s investigate this challenge in the case of the previous example

- In a democratic setting, the regulatory function θ is likely to be a function of the set of individual preferences at equilibrium.

- Let’s see more precisely what it means on our previous example

Let's investigate this challenge in the case of the previous example

- The governance should define a function $G(S)$, which gives the regulation as a function of the output S for all agents at equilibrium

- For example, suppose that (in the previous framework) the governance has decided that :
  - The function $\theta(x)$ is a linear function $\theta(x) = Ax$,
  - Each agents a can express his own preference $f(a)$
  - It has been agreed that the parameter A will be : $A = \int f(a) dm_0(a)$

Let's investigate this challenge in the case of the previous example

- The solver of the equilibrium problem, gives the value at equilibrium
- $v(a, A) = Max\ P_0$ (individual max, as part of the equilibrium process)

- The value at equilibrium $v(a, A)$ is a function of agent's a individual parameters, and of regulation parameter A

- The (non-strategic) optimal value of A for agent a is
- $f(a) = Arg.Max_A\ v(a, A)$

- This gives a function f , and close the governance loop in this example

Strategic behaviors and vote

- Again : we have now solvers for such fix points problems

- Hence : we can assess the previous governance scheme

- But this simple example enlight one of the issue of the governance challenge :
- Agents may have incentives to behave strategically
- And to declare a value $f(a)$ based on sophisticated strategies
- (this challenge is part of vote theory)

- Hence, the assessing some governance scheme
- is often a more sophisticated challenge than just the previous fix point framework

Conclusion

- We now have a comprehensive suite of quantitative mathematical tools
- designed for modeling and decision support,
- aimed at assessing the equilibrium
- generated by a regulatory framework.

- Furthermore, we can quantitatively explore the consequences of various governance frameworks, thereby enhancing our understanding of potential conflicts
- But of course, implementation of these new quantitative modeling capacities requires tough time-consuming work in each specific case